BRAIN
ORIGINAL ARTICLE

Continuous manipulation of mental representations is compromised in cerebellar degeneration

Samuel D. McDougle,[1,†] Jonathan S. Tsay,[2,3,†] Benjamin Pitt,[2] Maedbh King,[2] William Saban,[2] Jordan A. Taylor[4,5] and Richard B. Ivry[2,3]

[†]These authors contributed equally to this work.

We introduce a novel perspective on how the cerebellum might contribute to cognition, hypothesizing that this structure supports dynamic transformations of mental representations. In support of this hypothesis, we report a series of neuropsychological experiments comparing the performance of individuals with degenerative cerebellar disorders on tasks that either entail continuous, movement-like mental operations or more discrete mental operations. In the domain of visual cognition, the cerebellar disorders group exhibited an impaired rate of mental rotation, an operation hypothesized to require the continuous manipulation of a visual representation. In contrast, the cerebellar disorders group showed a normal processing rate when scanning items in visual working memory, an operation hypothesized to require the maintenance and retrieval of remembered items. In the domain of mathematical cognition, the cerebellar disorders group was impaired at single-digit addition, an operation hypothesized to primarily require iterative manipulations along a mental number-line; this group was not impaired on arithmetic tasks linked to memory retrieval (e.g. single-digit multiplication). These results, obtained in tasks from two disparate domains, point to a potential constraint on the contribution of the cerebellum to cognitive tasks. Paralleling its role in motor control, the cerebellum may be essential for coordinating dynamic, movement-like transformations in a mental workspace.

1 Department of Psychology, Yale University, New Haven, CT 06511, USA
2 Department of Psychology, University of California, Berkeley, Berkeley, CA 94704, USA
3 Helen Wills Neuroscience Institute, University of California, Berkeley, Berkeley, CA 94704, USA
4 Department of Psychology, Princeton University, Princeton, NJ 08540, USA
5 Princeton Neuroscience Institute, Princeton University, Princeton, NJ 08540, USA

Correspondence to: Samuel D. McDougle
314D Sheffield-Sterling-Strathcona Hall
Yale University, New Haven CT 06520, USA
E-mail: samuel.mcdougle@yale.edu

Correspondence may also be addressed to: Jonathan S. Tsay
Berkeley Way West, 3rd Floor
University of California, Berkeley, Berkeley CA 94704
E-mail: xiaotsay2015@berkeley.edu

Keywords: cerebellum; spinocerebellar ataxia; mental operations; mental imagery

Received May 04, 2021. Revised January 11, 2022. Accepted February 05, 2022. Advance access publication February 24, 2022

Introduction

The functional domain of the cerebellum extends beyond sensorimotor control.[1,2] Anatomical studies have revealed broad patterns of connectivity between the cerebellum and most of the cerebral cortex in humans and non-human primates, including prominent reciprocal connections between the cerebellum and prefrontal cortex.[3–5] Work in rodent models has linked cerebellar activity to a variety of surprising non-motor functions, such as reward processing, decision-making and social interaction.[6–8] Human functional neuroimaging studies have revealed consistent cerebellar activation patterns unrelated to overt movement,[9–11] and neuropsychological studies have identified a large set of non-motor tasks on which individuals with cerebellar pathology are impaired.[2]

The most common neuropsychological sequelae in individuals with degenerative cerebellar disorders (CD) are impairments in cognitive control,[12] including visuospatial cognition,[13] working memory[14,15] and abstract reasoning.[16] Echoing the loss of motor coordination observed in these patients (movement dysmetria), the phrase 'dysmetria of thought' has been used to summarize heterogeneous cognitive symptoms associated with CD.[17] This phrase reflects the idea that core mental functions, such as perception and memory, are spared in cerebellar pathology, but the ability to manipulate mental representations in a coordinated manner is compromised.

A number of computational hypotheses have been put forward based on the idea that cerebellar contributions to motor control may generalize to the cognitive domain.[18,19] In the motor domain, computational accounts of cerebellar function have emphasized the importance of this structure in anticipating future states over a relatively short temporal window.[20–22] For example, in coordinating action, the cerebellum is hypothesized to continuously generate predictions related to the sensory consequences of an impending movement.[23] These predictions are compared with incoming sensory feedback, with the difference between the two serving as an error signal to calibrate the sensorimotor system.[24]

Extending this idea to cognition has motivated hypotheses about a cerebellar role in predictive processing that generalizes beyond motor control. For instance, it has been suggested that the cerebellum predicts future cognitive states via mental simulation.[19] However, given that prediction is a general feature of brain function,[25] a key challenge is to specify constraints on how the cerebellum may contribute to prediction. In terms of motor control, one such constraint arises from the fact that movement entails the continuous transformation of the body: A movement goal may be couched in terms of a desired end state, with the motor system optimized to transform the body's initial state to the desired end state in a smooth manner.

Drawing an analogy to how the cerebellum aids in continuous transformations of the body during motor control, we asked whether the cerebellum also supports the continuous, coordinated transformation of mental representations during non-motor tasks. To test this idea, we draw on the cognitive psychology literature that places mental operations on a spectrum between those that emphasize continuous versus discrete transformations.[26] We hypothesize that the cerebellum might be especially important in tasks involving continuous, movement-like transformations of internal representations.

We tested the continuity hypothesis in two disparate non-motor domains, visual cognition and arithmetic. For visual cognition, an extensive body of behavioural and physiological research provides strong evidence that operations involved in visual imagery and mental rotation require continuous representational transformations.[27] For example, in mental rotation, the manipulation of a visual representation to facilitate object recognition entails movement through intermediate representational states.[27,28] This continuity constraint is strengthened by neurophysiological evidence, where, for instance, movement planning elicits the continuous transformation of a population vector in motor cortex.[29,30] We hypothesized that individuals with CD would exhibit a disrupted (slowed) rate of mental rotation, which would be consistent with neuroimaging studies showing prominent cerebellar activation during mental rotation tasks.[10,31]

As a 'non-continuous' control, we opted to use two visual memory search tasks in which participants compare a probe stimulus to a set of discrete items maintained in memory.[30,32] While recognizing that there are several differences between mental rotation and memory search, we chose the latter as a control because the rate-limiting factor in memory search does not require a continuous transformation of a single representation but rather a retrieval operation on items held in working memory. We predicted that the CD group would show a normal rate of search through working memory.

As a test of generality, we evaluated the continuity hypothesis in a second domain, mental arithmetic, leveraging an analogous contrast between tasks that entail continuous transformations versus discrete retrieval. Multiple lines of behavioural and neuroimaging research posit the use of a continuous, spatialized representation that supports basic mathematical operations such as magnitude comparison and simple addition (i.e. a mental number line).[33–38] In simple arithmetic tasks, reaction times (RTs) increase in a linear manner with the magnitude of the operands,[39] and error patterns exhibit 'operational momentum' while internally 'moving' along the mental number line. For example, people tend to overestimate solutions to addition problems and underestimate solutions to subtraction problems.[40,41] Similarly, patients with spatial neglect from right hemisphere lesions show systematic biases when mentally bisecting numerical intervals (e.g. stating 16 as midpoint between 12 and 18), echoing the rightward bias they exhibit in bisecting physical lines.[40] These findings support the notion that simple addition primarily entails a continuous mental transformation. As such, we predicted that individuals with CD would be impaired on a numeric verification task involving the addition of two single digit numbers, showing a steeper magnitude effect relative to controls.

For our non-continuous control tasks, we selected two numeric operations that have been demonstrated to rely on rote retrieval: Addition problems involving the same number and single-digit multiplication. For the former, RTs are largely independent of the magnitude of the operand.[42,43] For multiplication, RT increases with magnitude, but the increase is attributed to a look-up operation mediated by problem frequency.[44–48] To the extent that identity addition problems and multiplication problems are memorized, solving them should not require a continuous transformation, but rather discrete sampling of information from memory.[37,46–48] We thus predicted that performance on identity addition and multiplication would be similar in the CD and control groups.

For each domain, we conducted two experiments, with the task involving a continuous transformation repeated in each experiment and the control task varied to test alternative hypotheses. By using different groups of participants for each experiment within a domain, we conducted a replication study within each domain

and assessed generalization across domains, providing a comprehensive test of the continuity hypothesis.

Participants and methods

Participants

Adult participants diagnosed with a CD of genetic or unknown aetiology (total $n = 48$) and neurologically healthy controls (total $n = 49$) participated in the study in exchange for monetary compensation ($20 per hour). Each test session lasted approximately 2 h, including time for obtaining informed consent, medical history, performing the neuropsychological (all participants) and neurological (for CD only) evaluations and conducting the experimental tasks. All participants completed a pair of experimental tasks associated with their assigned experiment (see below). Most of the participants (CD = 40, controls = 42) completed a third sensorimotor control task unrelated to the present report. The other participants (CD = 8, controls = 7) completed two of the experiments (specifically Experiments 1b and 2b) in a single session (and thus performed a total of four experimental tasks). Each task took between 15–30 min, and breaks were provided between tasks. The protocol was approved by the institutional review boards at Princeton University and the University of California, Berkeley.

All participants were screened for general cognitive deficits using the Montreal Cognitive Assessment (MoCA). Inclusion required that the participant achieve a score above 20 on the MoCA 30-point scale, a boundary that is more liberal than the typical cutoff chosen to exclude individuals with mild cognitive impairments (e.g. <26). We opted to use a liberal cutoff given our general interest in the contribution of the cerebellum to cognition and specific focus in testing the continuity hypothesis. Nonetheless, given the liberal inclusion criterion, we included MoCA scores (and age) as covariates in our analyses.

Inclusion in the CD group was based on genetic confirmation of spinocerebellar atrophy (SCA, see Supplementary Table 1) or a clinical diagnosis of ataxia. Twenty-six of the 48 individuals had an identified subtype (SCA1: 4; SCA2: 2; SCA3: 4; SCA5: 1; SCA6: 10; SCA8: 1; SCA15: 1; SCA28: 1; AOA2: 2), and the 10 CD participants with confirmed SCA6 were related. The other 22 individuals in the CD group were classified as having sporadic adult-onset ataxia of unknown aetiology (SAOA), with no family history of ataxia or confirmed genetic classification. Based on a prescreening interview, we excluded candidate participants who were diagnosed with ataxia but had no MRI evidence of cerebellar degeneration, lacked genetic confirmation of SCA or did not report a family history of SCA.

All CD participants were clinically evaluated at the time of testing with the Scale for Assessment and Rating of Ataxia (SARA[49]). Scores ranged from 2 (mild motor impairments) to 26 (severe motor impairments). We did not formally evaluate the CD participants for extracerebellar signs, although none of the participants had visual evidence of resting tremor, dystonia or bradykinesia. For each experiment, we performed *post hoc* analyses limited to individuals with SCA6 and SAOA, variants in which the pathology is mostly limited to the cerebellum.[50,51] In doing so, we excluded individuals with SCA1, SCA2, SCA3, SCA5, SCA8, SCA15, SCA28 and AOA2, given that these variants are associated with extracerebellar pathology.[50,52]

Control participants were recruited to provide a match to the clinical sample in terms of age, MoCA and years of education (Table 1 and Supplementary Table 2). There were three exceptions: (i) we neglected to collect years of education in Experiment 1a—an oversight corrected in the other experiments (Experiments 1b, 2a and 2b); (ii) by emphasizing matches based on education in Experiment 1b, we ended up with a CD sample that was slightly but significantly older than the control group; and (iii) by emphasizing matches in other categories, the MoCA scores were significantly lower in the CD group compared with the control group in Experiment 2b. As noted above, MoCA and age were included as covariates in the primary analyses of all the experiments.

Apparatus and procedure overview

For all experiments, the stimuli were displayed and responses recorded on a laptop computer (MacBook Pro, Apple) using the psychophysics toolbox package[53] for MATLAB (MathWorks). Participants were seated a comfortable distance from the screen (viewing distance ~40 cm). Responses were made with the index and ring fingers of the right hand on the computer keyboard.

Experimental tasks

Experiment 1

Experiments 1a and b were designed to evaluate the continuity hypothesis in the domain of visual cognition. Each involved two conditions, an experimental condition hypothesized to entail a continuous operation and a control condition, hypothesized to entail a non-continuous, or discrete operation. In both Experiments 1a and b, the same mental rotation task was used for the continuous condition. It was paired with a visual working memory task in Experiment 1a and a visuospatial working memory task in Experiment 1b. The continuous and control tasks were tested in separate experimental blocks within a single session, with the order counterbalanced. Each task took approximately 25 min to complete, and participants were given a 10 min break between conditions.

Mental rotation (Experiments 1a and b)

Following the basic design described by Shephard and Metzler,[28] participants judged if a visual stimulus was normal ('R') or mirror-reflected ('Я'; Fig. 1). Eight capitalized sans-serif (Helvetica font) letter stimuli were used, consisting of normal and reflected versions of the letters F, G, J and R.[54] Letter stimuli were white and presented on a black background. To minimize eye movements while maintaining stimulus legibility, the stimuli were modestly sized (~4 cm^2), of high contrast and presented at a central location on the monitor.

Participants were instructed to press the right arrow key with their right ring finger when the stimulus (if viewed or imagined in an upright orientation) was in standard form and the left arrow key with their right index finger if the stimulus was mirror reflected. The stimulus was presented in the standard upright orientation (0°, baseline condition), or rotated, using one of 10 angles drawn from the following set: −135°, −105°, −75°, −45°, −15°, 15°, 45°, 75°, 105° and 135°. The stimulus remained visible until the response, or for a maximum of 5 s, whichever came first. After the response, feedback was shown above the letter for 1 s. On correct trials, the word 'correct' was displayed in green font. On incorrect trials, the word 'incorrect' was displayed in red font. Participants were instructed to respond quickly, while maintaining a high level of accuracy. If a response was not made within 5 s, the message 'too slow' was displayed in red font. Following a 1 s feedback interval, the display was replaced by a white fixation cross (0.9 cm^2) that remained visible for a 2 s inter-trial interval.

Participants performed 18 trials of each rotation size and sign, intermixed with 36 no-rotation baseline trials, for a total of

Table 1 Demographic summary of CD and control participants across all four experiments

	Age		MoCA		Years of education		Sex		SARA
	CD	Control	CD	Control	CD	Control	CD	Control	CD
Exp. 1a	49.8 (37–70)	50.2 (37–70)	28 (26–29)	29 (27–30)	Not collected		5 F, 7 M	5 F, 7 M	11.8 (5–22)
Exp. 1b	57.2* (32–71)	50.7* (33–73)	25 (21–28)	27 (22–29)	15.8 (12–22)	16.4 (13–20)	11 F, 3 M	10 F, 4 M	11.3 (2–25)
Exp. 2a	53.3 (33–68)	56.5 (32–74)	27 (20–30)	28 (25–30)	16.9 (12–24)	17.7 (16–22)	10 F, 5 M	7 F, 8 M	13.7 (5–26)
Exp. 2b	56.5 (32–77)	58.3 (34–73)	25* (21–28)	27* (24–30)	15.4 (12–20)	16.9 (15–20)	11 F, 4 M	11 F, 4 M	11.5 (2–26)

Exp. = Experiment; F = female; M = male.
*Significant difference ($P < 0.05$) between groups.

216 trials. Stimuli were presented in a random order, with an equal number of normal and reflected presentations of each stimulus at each rotation sign and magnitude. Prior to the start of the experimental block, the participants performed five practice trials to ensure that they understood the task instructions and were comfortably positioned to respond on the keyboard.

Visual memory search: Experiment 1a

As a control task in Experiment 1a, we employed a variant of the 'memory scanning' task introduced by Sternberg.[32] On each trial, participants viewed a brief sequence of visual stimuli and, after a maintenance period, judged whether a probe stimulus was a member of the previous sequence (match) or not (non-match). The stimuli consisted of 30 colourful fractal-like patterns, generated using the randomization function of ArtMatic Pro (www.artmatic.com). The images were cropped to be square-shaped and were matched in size to the mental rotation stimuli (4 cm^2).

The memory set was presented sequentially with each fractal image in the set displayed in the centre of the screen for 1 s (with no inter-stimulus interval). To vary the working memory load across trials, the number of fractals in a set ranged from one to five items. After the sequence terminated, the screen was blanked for a maintenance period of 3 s. A probe stimulus was then presented. Participants were instructed to press the right arrow key with the right ring figure in the event of a match and the left arrow with their right index finger in the event of a non-match. The probe remained visible until the response or until 5 s had elapsed. Feedback ('correct', 'incorrect', 'too slow') was displayed above the probe stimulus for 1 s after the response was made. The display was then replaced by a white fixation cross for a 2 s inter-trial interval.

In 50% of the trials, the probe matched one of the items in the sequence, and in the other 50% of the trials, the probe did not match any of the items. Twenty trials at each set size were presented (10 match, 10 non-match) in a random order for a total of 100 trials. Participants completed five practice trials at the start of the experiment.

Spatial visual memory search: Experiment 1b

A spatial working memory task was employed for the control condition in Experiment 1b, adopted from a task introduced by Georgopoulos and Pellizzer.[30] In each trial, a sequence of red circles (diameter 1.2 cm) was displayed on a white ring (radius 7 cm). A circle could be presented at any location from 0°–345° (at multiples of 15°), with the constraint that no location be repeated in a given sequence. Each stimulus was presented for 800 ms, with no time gap between successive targets. The number of stimuli in the sequence ranged from two to five items. Following the offset of the last item in the sequence, the white ring remained on the screen for a maintenance period of 2 s, after which a probe stimulus was shown. The probe always appeared in one of the positions previously shown in the sequence. Participants were instructed to press the number on the keyboard corresponding to the ordinal position of the probe within the sequence (i.e. '1' key if location of first item, '2' key if location of second item, etc.). The probe remained visible until the response, and feedback was presented for 1 s following the response. During the 2 s inter-trial interval, the white ring remained visible.

Each set size (i.e. sequence length 2–5) was presented 30 times in a randomized order, for a total of 120 trials. Within each set size, probe positions were sampled uniformly between the first and the second-to-last position; for example, if the set size was five, the probe location could match the location of the first, second, third or fourth target in the sequence. Except for a set size of two, we chose not to include trials probing the terminal position given the asymmetrically large RT benefit for this position observed in pilot testing. The task started with five practice trials to ensure that participants understood the instructions.

Experiment 2

Experiments 2a and b evaluated the continuity hypothesis in the domain of mathematical cognition. Experiment 2a consisted of only addition problems where the continuous condition was composed of equations in which the two operands were non-identical, and the control condition was composed of equations in which the two operands were identical. These two types of equations were intermixed in a single block of trials that took approximately 25 min to complete. Both addition and multiplication problems were tested in Experiment 2b, with the latter providing a second control condition. To minimize task-switching costs, the two types of mathematical operations were tested in separate blocks, each lasting approximately 25 min (plus the 10 min break), with the order counterbalanced across individuals.

Addition verification: Experiments 2a and b

In each trial, participants indicated if an addition equation, composed of two single digit operands and a sum, was either true or false (Fig. 4).[43] The continuous condition consisted of trials in which the operands were non-identical (e.g. 4 + 7 = 11); the control condition consisted of trials in which the operands were identical (e.g. 6 + 6 = 12). Equations were white and presented in the centre of the screen on a grey background. To minimize the necessity of making large saccades while maintaining stimulus legibility, the total length of the equation was modestly sized (4 cm). The equations were presented in a standard format (e.g. 3 + 7 = 11) with the sum always having two digits (e.g. '09' if the indicated sum was 9)

to reduce the use of heuristics (e.g. recognizing that 3 + 1 could not be a two digit sum).

Equations were drawn from a set of 36 single digit equations. Equations in Experiment 2a were composed of all unique combinations of operands between 3 and 8. Equations in Experiment 2b were composed of all unique combinations of operands 3, 4, 6, 7, 8 and 9. Operands less than 3 were removed to limit the number of combinations where magnitude effects were modest.[43] We also excluded the operand 9 in Experiment 2a to further limit the number of combinations; we replaced the 5 with 9 in Experiment 2b, given concerns that multiplying by 5 (see Experiment 2b control condition below) would be easier than other two-digit multiplication problems. Each equation was presented eight times, consisting of four true responses and four false responses. True responses had one unique equation with the correct sum provided (e.g. 3 + 4 = 7). For the false equations, there were four distinct erroneous sums for each equation, with the presented sum different from the actual sum by either ±1 (e.g. 3 + 4 = 8; 3 + 4 = 6) or ±2 (e.g. 3 + 4 = 9; 3 + 4 = 5).

There were 288 addition trials in total, with the equations with identical and non-identical operands randomly interleaved. The trial sequence was subject to three constraints to minimize the effects of numerical and response priming on RT[55]: (i) If answered correctly, the same response would not occur more than three times in succession; (ii) Consecutive problems could not share identical operands; and (iii) Consecutive problems could not share the same solution. Note that this third constraint limits repetition of equations with the actual true sum, not the displayed sum, which could either be a true or false sum. The entire block took approximately 25 min.

Multiplication verification (Experiment 2b)

Participants completed a second block of trials in Experiment 2b, in which they performed the verification task on multiplication equations. This condition was added to provide a second control condition given evidence indicating that computing the product of two single-digit numbers is primarily based on referencing rote memory.[42] The method for multiplication was identical to that used in addition, including the use of the same set of operands (3, 4, 6, 7, 8 and 9). Equations with erroneous products also had four variants, here created by adding ±1 to either the first operand (e.g. 8 × 3 = 27 or 8 × 3 = 21) or ±1 from the second operand (e.g. 8 × 3 = 16 or 8 × 3 = 32). There was a total of 288 trials consisting of equations with identical and non-identical operands, which were analysed separately as in addition. The order of the addition and multiplication tasks in Experiment 2b was counterbalanced across individuals.

Data analyses

Trials associated with extreme outlier RTs [>±3.5 standard deviations (SD) from the participant's mean] were removed prior to the analysis of the RT data. A similar number of trials were removed for CD and control groups (Supplementary Table 3; <3% per group for all tasks).

Across all four experiments, we used a linear mixed effect model framework (R function: lmer) to analyse our dependent variables (DV: RT or accuracy), with Group, Task and the task-relevant independent variable (IV: rotation magnitude, set size, max operand) as the fixed effects of interest and Subject ID (S) as a random effect:

$$DV \sim Group \times Task \times IV + Covariates + (1|S) \quad (1)$$

We opted to use a linear mixed effect model for three reasons: (i) this model is considered robust even if the normality assumption is violated[56] (as is typically the case with RT); (ii) a linear mixed effect model is amenable to our planned *post hoc* analyses (e.g. differences in RT slopes between groups) and analyses involving additional covariates (e.g. MoCA, age); and (iii) by including subject as a random effect, linear mixed effect models account for within-subject variability when assessing for a three-way interaction amongst Group, Task and the task-relevant independent variable, providing an ideal analysis of the core hypothesis for all four experiments.

We employed F-tests with the Satterthwaite method to evaluate whether the coefficients (i.e. beta values) obtained from the linear mixed effects model were significant (R function: ANOVA). Pairwise *post hoc* *t*-tests (two-tailed) were used to compare CD and control RT slopes (R function: emtrends). *P*-values were adjusted for multiple comparisons using the Tukey method and 95% confidence intervals are reported in squared brackets.

For the CD group, we also asked if the rate of computation was related to motor symptom severity. To assess this, we extracted individual RT slopes via linear regression (R function: lm) in each task and examined the correlation of these values with SARA scores (R function: cor.test). We note that because SARA was only assessed in the CD group, we did not include this variable as a covariate in the linear mixed effect model.

Visual cognition tasks (Experiments 1a and b)

The three visual cognition tasks were selected because each has been shown to produce a linear function relating the main independent variable—absolute rotation magnitude in mental rotation or set size in the two working memory search tasks—to RT. Our hypothesis centered on the three-way interaction of Group, Task and the task-relevant independent variable on RT. We predicted a larger slope for the CD group compared with the controls on mental rotation rate but no group difference on memory search rate. To directly compare group differences in RT across tasks with different independent variables (rotation magnitude for mental rotation; set size for memory search), we z-scored the independent variable within each task. We note that the beta coefficients reported in the 'Results' section were rescaled to their original units for ease of interpretation.

Mathematical cognition tasks

With the exception of problems with identical operands, addition and multiplication verification tasks involving two single-digit operands have been shown to produce an increase in RT as a function of magnitude, with magnitude defined in a range of ways (max operand, min operand, first operand, second operand or the solution).[43,57,58] We opted to use max operand as the independent variable for both the addition and multiplication tasks in the present study, although the pattern of results was not affected when magnitude was operationalized with any of the four other definitions.

Unlike the visual cognition tasks, the mathematical cognition tasks all involved the same independent variable, max operand. Therefore, we could directly compare the performance on the continuous versus non-continuous control tasks within a linear mixed effect model without requiring a z-score transformation. We again predicted a three-way interaction, expecting that the CD group would show a steeper slope than the controls on addition equations involving non-identical operands, but no slope differences between groups for addition equations with identical operands or multiplication equations.

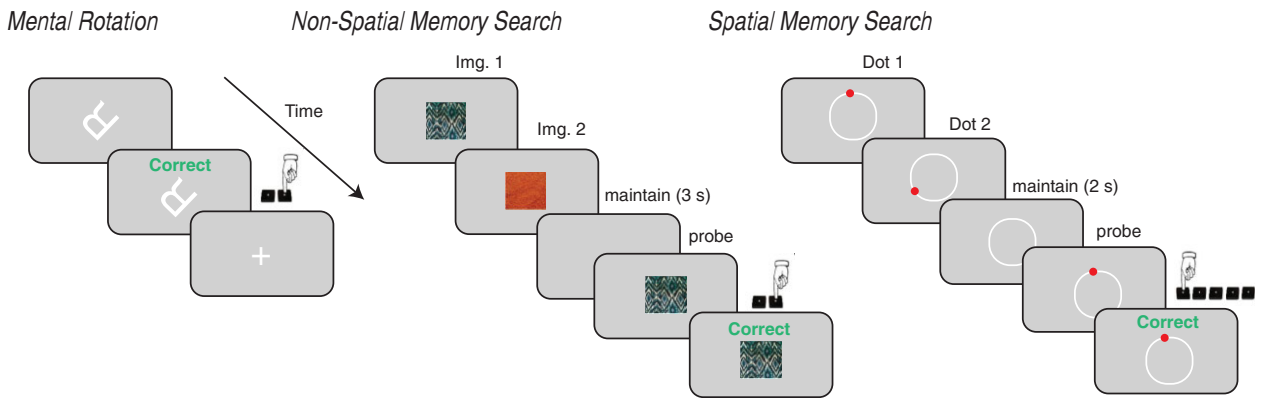

Figure 1 Visual cognition tasks employed in Experiments 1a and b. In the mental rotation experiment, participants judged if a letter stimulus was normal (e.g. 'R', right key press) or mirror-reflected (e.g. 'Я', left key press). On most trials, the stimulus was rotated relative to the upright orientation (depicted example involves 135° rotation). The same mental rotation task was used in Experiments 1a and b for the continuous condition. Two memory search tasks were used for the non-continuous, control conditions. In the visual memory search task (Experiment 1a), a sequence of stimuli (1–5 images) was presented (1 s per image). After a maintenance period (3 s), a probe stimulus appeared, and the participant judged whether it was a member of the memory set (right key) or not (left key). Sequences varied in length from one to five items. In the visuospatial working memory task (Experiment 1b), a sequence of circles (2–5 items) was presented at random locations on a ring (1 s per target). After a maintenance period (2 s), a probe stimulus was presented, and the participant indicated the ordinal position of the probe. Responses in all tasks were followed by feedback (1 s), and a 2 s inter-trial interval.

Data availability

The data that support the findings of this study are available from the corresponding authors upon request.

Results

Experiment 1a

In Experiment 1, we tested the prediction that individuals with cerebellar degeneration would be selectively impaired on a visual cognition task that required the continuous transformation of a mental representation. For the continuous task (Fig. 1), we employed a classic mental rotation task.[28] Participants ($n = 12$ CD and $n = 12$ control) judged if a visual letter stimulus was normal ('R') or mirror-reflected ('Я'), where the stimulus was rotated by a particular degree on each trial: −135°, −105°, −75°, −45°, −15°, 0°, 15°, 45°, 75°, 105° and 135°. For the non-continuous task, we employed a classic memory search task[32] that requires the retrieval of discrete visual representations held in working memory. Participants viewed a single or sequence of abstract, visual fractal stimuli (set size 1–5), and, after a brief maintenance period, were asked to judge whether a probe stimulus was a member of the set (match) or not (non-match).

Reaction time effects

We first considered general features of task performance. There was no effect of task [$F(1,6782) = 1.2$, $P = 0.22$, $\beta = 17.2$ (−10.1, 44.5)], indicating that average RTs were comparable in the mental rotation and memory search tasks. Considering only correct trials, the CD group responded slower than the control group on both mental rotation [CD: 1361 ± 600 ms; Control: 869 ± 326 ms; $t(20) = 3.9$, $P < 0.001$, $\beta = 491$ (242.1, 740.0)] and memory search tasks [CD: 1385 ± 560; Control: 889 ± 348; $t(21) = 4.0$, $P < 0.001$, $\beta = 513$ (242.1, 740.0)]. The slower RTs in the CD group may, in part, reflect motor deficits associated with ataxia. However, the large group differences may also reflect additional factors, an issue we return to in the 'Discussion' section. RT did not vary with age [$F(1,20) = 0.4$, $P = 0.51$, $\beta = 3.1$ (−5.5, 11.7)] or MoCA score [$F(1,20) = 0.2$, $P = 0.67$, $\beta = 22.4$ (−74.3, 119.1)].

We next assessed whether both groups exhibited the RT effect classically associated with these two tasks, namely that RTs increase with the experimentally titrated independent variable—rotation magnitude for mental rotation and set size for visual memory search. Indeed, for both tasks, RTs increased with the size of the independent variable [$F(1,6782) = 526.5$, $P < 0.001$, $\beta = 114.4$ (99.4, 129.5)], replicating classic results in the literature (Fig. 2). Rates of computation (RT slopes) were not related to motor severity in the CD group (mental rotation: $R = -0.34$, $P = 0.28$; memory search: $R = -0.46$, $P = 0.13$).

Turning to the main question at hand, we used a linear mixed effect model to ask whether the rate of computation differed between groups on the two tasks. Critically, there was a significant three-way interaction of Group, Task and each independent variable [$F(1,6780) = 20.6$, $P < 0.001$, $\beta = -92.1$ (−131.8, −52.3)]. This three-way interaction was driven by the mental rotation RT slopes being selectively elevated in the CD group compared to the controls [$t(6782) = 5.53$, $P < 0.001$, $\beta = 1.2$ (0.8, 1.6)]; the CD group had a rate of mental rotation of 3.4 ± 0.2 ms/°, ~1.5-fold higher than the rate of mental rotation in the control group (2.2 ± 0.2 ms/°). This finding suggests that cerebellar degeneration disrupts the rate of mental rotation, independent of any motor or generic cognitive impairments.

At an individual level, seven of the 12 CD participants had slower mental rotation rates than the slowest control. Interestingly, two individuals in the CD group showed the fastest mental rotation rates (i.e. lowest slopes) overall (Fig. 2B, two lightest dots in the CD group). These two participants also exhibited the highest error rates, responding below 75% accuracy (Supplementary Fig. 1A). Although speculative, difficulty in performing mental rotation may have led these individuals to adopt an alternate strategy, perhaps making intuited responses based only on the presented orientation of the stimuli. If these two participants were excluded from the slope analysis, RT slopes remained significantly elevated in the CD versus control group [$t(6323) = -7.15$, $P < 0.001$, $\beta = 1.6$ (1.2, 2.3)].

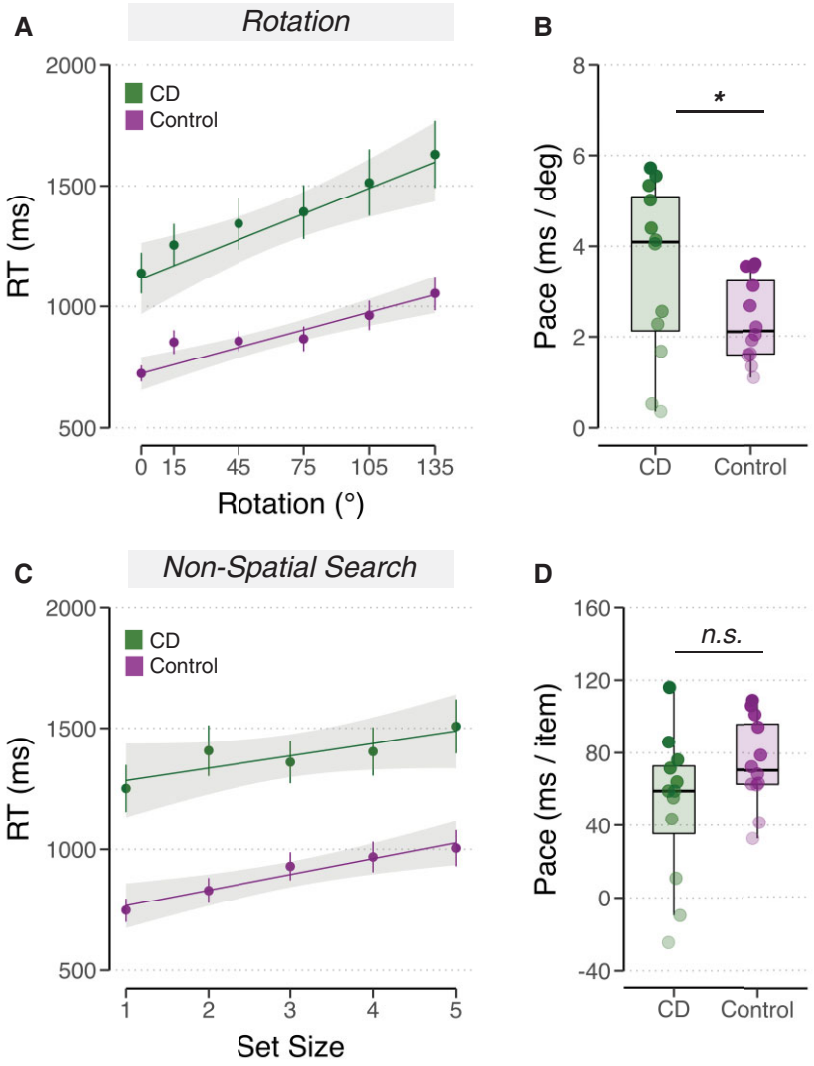

Figure 2 Reaction time analysis for Experiment 1a. Cerebellar degeneration is associated with a slower rate of mental rotation but does not impact the rate of search through visual working memory. (**A**) Median RT as a function of stimulus orientation in the mental rotation task for the CD group and the control group. (**B**) Estimated rate of rotation from the regression analysis (slope of RT function). (**C**) Median RT as a function of set size in the visual memory search task. (**D**) Estimated search rate from the regression analysis. Mean regression lines are displayed in **A** and **C**. Shaded error bars denote 1 SEM. n.s. = not significant. *$P < 0.05$.

In contrast to the group differences in mental rotation, RT slopes did not differ between groups in the memory search task [$t(6782) = 5.53$, $P = 0.27$, $\beta = 19.0$ (−11.1, 31.1)]. The average RT slopes were 45.6 ± 7.8 ms/item in the CD group and 65.0 ± 7.3 ms/item in the control group, indicating that the two groups had similar rates of memory retrieval. Taken together, our results revealed that the CD group exhibited a specific impairment on the mental rotation task, which is associated with a continuous operation, rather than a general deficit in visual cognition.

Accuracy effects

Although our predictions focused on RT, we also examined accuracy on the two tasks (Supplementary Fig. 1A). Overall accuracy in both groups was higher in the mental rotation task (93.2%, SD = 7.2) compared with the memory search task (85.7%, SD = 6.7) [$F(1,7554) = 118.0$, $P < 0.001$, $\beta = -0.05$ (−0.07, −0.03)]. Accuracy decreased as the computational demands of each task increased, either due to an increase in rotation angle or memory load [$F(1,7554) = 158.5$, $P < 0.001$, $\beta = -0.03$ (−0.04, −0.02)]. Accuracy was not significantly related to age [$F(1,20) = 1.1$, $P = 0.31$, $\beta = 0.00$ (0.00, 0.00)] or MoCA scores [$F(1,20) = 0.8$, $P = 0.38$, $\beta = 0.00$ (−0.01, 0.03)]. Overall, the CD group performed worse than the control group on the memory search task [$t(26) = 1.8$, $P = 0.002$, $\beta = -0.09$ (−0.14, −0.04)]. However, we did not observe a group difference on the mental rotation task [$t(21) = 1.8$, $P = 0.09$, $\beta = 0.05$ (−0.09, 0.00)].

It is important to consider whether the three-way interaction in RTs might arise from a difference in the trade-off between speed and accuracy: The inference that the rate of memory search is unaffected in CD would be problematic if accuracy worsened faster with set size in the CD group compared to the control group. To test this, we applied a linear mixed effect model on the accuracy data. The three-way interaction was not significant [$F(1,7554) = 0.1$, $P = 0.79$, $\beta = 0.00$ (−0.02, 0.03)]. Despite the absence of a three-way interaction, we also compared slopes between groups for each task separately, paralleling our analysis of the RT data. There were no group differences in the slopes of the accuracy functions in either the memory search task [$t(7554) = 0.31$, $P = 0.99$,

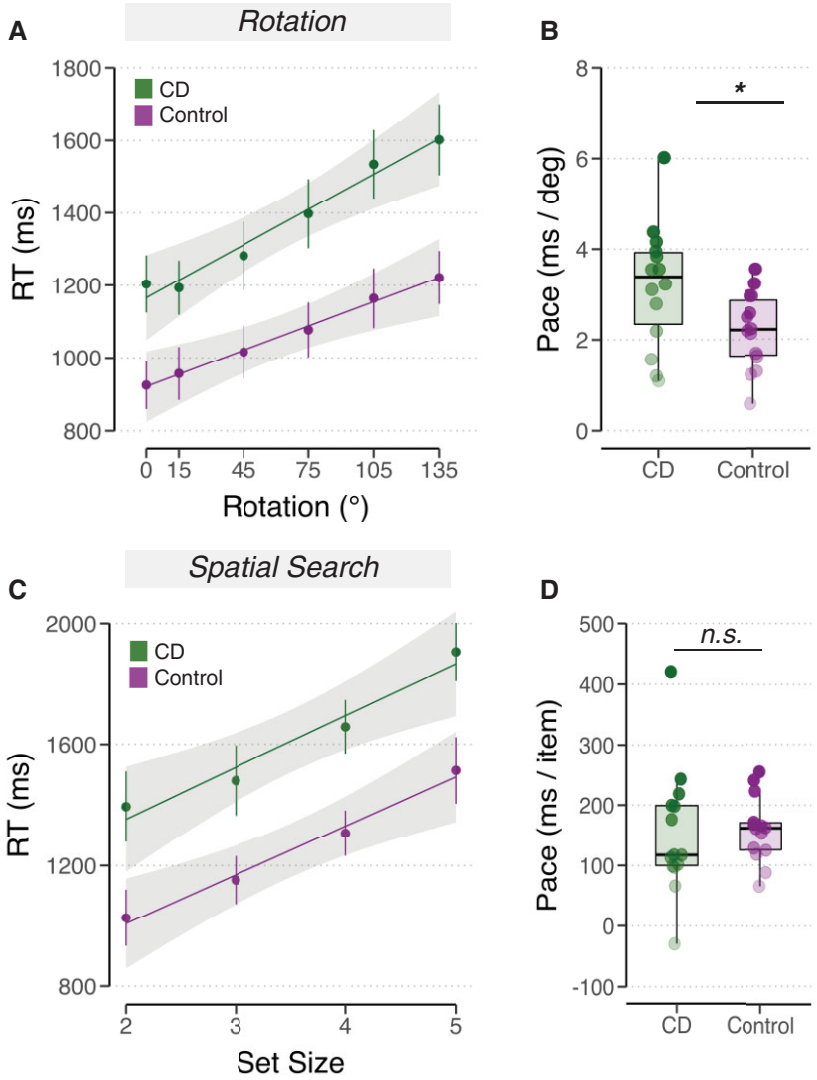

Figure 3 Reaction time analysis for Experiment 1b. Replication of selective impairment on the continuous transformation condition. In the mental rotation task (**A** and **B**), the CD group showed slower mental rotation speeds relative to controls. Mental rotation rates are plotted with the absolute rotation magnitude of the stimulus on the *x*-axis, and the median change in RTs for each rotation condition on the *y*-axis. In the search task (**C** and **D**), the two groups showed comparable spatial memory search speeds. The length of the test sequence (set size) is plotted on the *x*-axis, and the median change in RTs for each set size on the *y*-axis. Mean regression lines are displayed in **A** and **C**. Shaded error bars denote 1 SEM. n.s. = not significant. $^{*}P < 0.05$.

$\beta = 0.0$ (−0.02, 0.03)] or the mental rotation task [$t(7554) = 0.93$, $P = 0.79$, $\beta = 0.0$ (−0.01, 0.02)]. In summary, there was no indication that the CD impairment in accuracy became more pronounced, relative to controls, with larger rotations or increases in set size, arguing against a speed-accuracy tradeoff account of the task dissociation observed in the RT analysis. Underscoring this point, the rate of performance decline was also not associated to motor symptom severity (mental rotation: $R = 0.25$, $P = 0.43$; memory search: $R = -0.30$, $P = 0.35$).

Experiment 1b

The main factor in selecting the control task in Experiment 1a was to have an independent variable—set size—that would produce a parametric increase in RT, providing a rate measure to index an iterative mental operation performed on a set of discrete representations. However, we recognize that there are many differences between mental rotation and visual memory search. While many of these differences are factors thought to affect the intercept of the RT function (e.g. the decision boundary for making the two-choice speeded decision), one notable difference between the two tasks is that spatial representation is much more critical for the mental rotation task. Given the association of the cerebellum in spatial processing, be it for motor control or spatial cognition,[59] it is possible that the selective impairment on the mental rotation task reflects the demands on the cerebellum for spatial cognition rather than a selective role in facilitating continuous representational transformations.

To explore this possibility, we conducted a second experiment in which we paired the mental rotation task with a new control task chosen to tax visuospatial working memory (Fig. 1). Here, a sequence of circles was presented on a visual ring, and after a delay period a probe stimulus was displayed, where the position of the probe matched the position of a previously presented circle. The

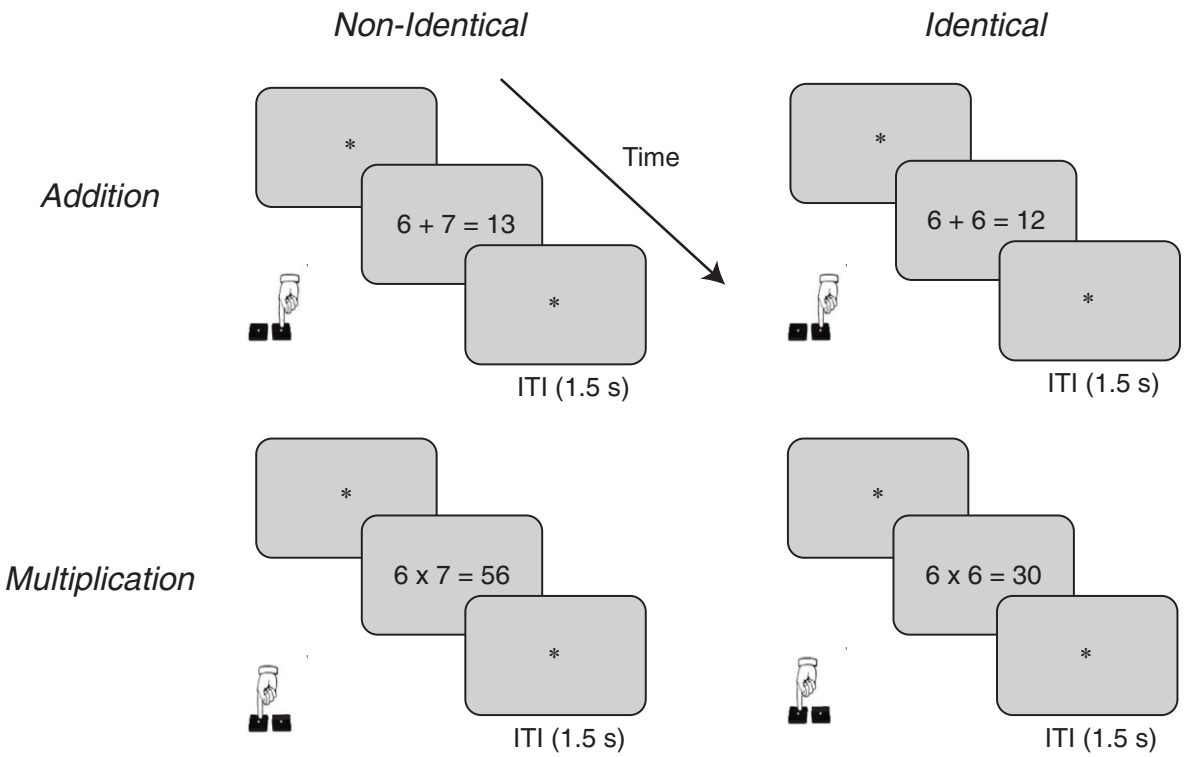

Figure 4 Mathematical cognition tasks employed in Experiments 2a and b. Participants made a speeded response to verify whether the equation was true or false. Addition problems (*top row*, tested in Experiments 2a and b) and multiplication problems (*bottom row*, tested only in Experiment 2b) involved either non-identical operands (*left column*) or identical operands (*right column*). Non-identical addition constitutes the continuous condition, whereas the remaining three conditions constitute non-continuous conditions.

participant indicated the ordinal position of the probe within the observed sequence by pressing one of five numbered keys. By varying the length of the sequence (2–5 locations), RT was expected to increase with set size, presumably reflecting a search through a set of discrete visual representations held in working memory. If the dissociation observed in Experiment 1a is related to the spatial processing demands associated with mental rotation rather than the continuous nature of the required mental transformation, we should expect to observe elevated RT slopes in the CD group on both the mental rotation task and the visuospatial working memory search task. In contrast, the continuity hypothesis predicts that the CD group would show an increase in RT slope only on the mental rotation task. Fourteen participants with CD and 14 control participants were tested in Experiment 1b, none of whom had participated in Experiment 1a.

Reaction time effects

Compared to the mental rotation task, RTs were on average 155 ms slower in the memory search task [$F(1,8113) = 241.2$, $P < 0.001$, $\beta = 154.8$ (123.0, 186.6)], indicating that memory search may be the harder of the two tasks. As such, the continuity hypothesis in this experiment predicts an impairment in the CD group in the 'easier' task. Consistent with Experiment 1a, RTs in the CD group were slower than the control group in both the mental rotation task [CD: 1365 ± 569 ms; Control: 1052 ± 527 ms; $t(24) = 2.0$, $P = 0.06$, $\beta = 242$ (0.9, 483.1)] and in the memory search task [CD: 1566 ± 607 ms; Control: 1184 ± 542 ms; $t(25) = 2.3$, $P = 0.03$, $\beta = 282$ (39.0, 525.0)]. RTs were neither affected by Age [$F(1,24) = 0.2$, $P = 0.64$, $\beta = 2.2$ (–6.5, 10.9)] nor MoCA score [$F(1,24) = 1.4$, $P = 0.25$, $\beta = –30.9$ (–80.1, 18.3)]. As expected, both tasks exhibited RTs that scaled with the independent variable [$F(1,8113) = 776.4$, $P < 0.001$, $\beta = 106.4$ (89.0, 123.8)]. The slope values for the two tasks were not related to motor symptom severity in the CD group (mental rotation: $R = 0.50$, $P = 0.17$; memory search: $R = –0.15$, $P = 0.61$).

We again observed a significant three-way interaction of Group, Task and each independent variable on RTs [$F(1,8113) = 4.2$, $P = 0.03$, $\beta = 47.9$ (–92.1, –3.7)]. Replicating Experiment 1a, the CD group showed a slower rate of mental rotation than the control group (Control: 2.0 ± 0.2, CD: 2.9 ± 0.2; Fig. 3A and B), confirmed by comparing the slope values relating RT to rotation angle [$t(8113) = 4.0$, $P = 0.004$, $\beta = 1.0$ (0.5, 1.4)]. This slope difference persists even when the one outlier in the CD group (i.e. rate of mental rotation >6; Fig. 3A right) was removed from the analysis [$t(7857) = 4.0$, $P < 0.001$, $\beta = 1.0$ (0.5, 1.5)]. In contrast, the CD and control groups exhibited similar slopes in the visuospatial memory search task [$t(8113) = –0.1$, $P = 1$, $\beta = 1.6$ (–26.9, 30.2); Control: 141.0 ± 10.5; CD: 142.5 ± 10.0; Fig. 3C and D)].

As a *post hoc* analysis, we compared mental rotation RT slopes across the two experiments. This between-experiment comparison showed no difference between both CD groups [$t(24) = 0.5$, $P = 0.64$ (–1.1, 1.7)] and control groups [$t(24) = 0.5$, $P = 0.62$ (–0.6, 0.9)], signalling a successful replication.

In summary, the RT data in Experiment 1b replicated the task dissociation observed in Experiment 1a: Degeneration of the cerebellum selectively disrupted the rate of computation on the visual cognition task that required a continuous transformation but had no effect on the rate of computation for operations required to evaluate a set of discrete representations in working memory. The results of Experiment 1b indicate that this dissociation cannot

be explained by a general involvement of the cerebellum in visuospatial processing.

Accuracy effects

In terms of accuracy (Supplementary Fig. 1B), performance was higher on the mental rotation task (94.2%, SD = 4.29) compared to the spatial memory search task (77.8%, SD = 11.26) [$F(1,9346)$ = 757.7, $P < 0.001$, $\beta = 0.2$ (0.20, 0.23)]. While this may reflect differences in task difficulty, the number of response options, and thus the chance performance level (two options in mental rotation, chance level = 50%; five in memory search, chance level = 50–20% depending on set size), differs between the two tasks in Experiment 1b. As expected, accuracy on both tasks worsened as the computational demands increased [$F(1,9346) = 821.2$, $P < 0.001$, $\beta = -0.15$ (–0.16, –0.13)]. Accuracy worsened with general cognitive status as indexed by the MoCA scores [$F(1,24) = 4.7$, $P = 0.04$, $\beta = 0.02$ (0.00, 0.02)] but was not affected by Age [$F(1,24) = 0.0$, $P = 0.84$, $\beta = 0.00$ (0.00, 0.00)]. As in Experiment 1a, there was no group difference in accuracy on the mental rotation task [$t(26) = -1.1$, $P = 0.27$, $\beta = 0.03$ (–0.01, 0.07)], but the CD group performed worse than the controls on the spatial memory search task [$t(30) = -4.3$, $P < 0.001$, $\beta = -0.11$ (–0.16, –0.06)].

To evaluate the possibility of a speed-accuracy tradeoff, we used a linear mixed effect model. As found for the accuracy data in Experiment 1a, the three-way interaction was not significant [$F(1,9346) = 0.1$, $P = 0.75$, $\beta = 0.00$ (–0.02, 0.02)], indicating that performance in both groups worsened at a similar rate for both tasks (Supplementary Fig. 1B). Nonetheless, we evaluated each task separately comparing the slope values between groups. The slopes were similar in both tasks [spatial memory search: $t(9346) = -0.52$, $P = 0.96$, $\beta = 0.0$ (–0.03, 0.01); mental rotation: $t(9346) = -1.24$, $P = 0.60$, $\beta = 0.0$ (–0.02, 0.01)]. In the CD group, the rate of performance decline was not related to motor symptom severity in either task ($R = -0.16$, $P = 0.57$; memory search: $R = -0.21$, $P = 0.48$).

Experiment 2: Assessing cerebellar contributions to mathematical cognition

The results of Experiments 1a and b are consistent with the continuity hypothesis, with the selective impairment on the rate of mental rotation hypothesized to reflect the cerebellum's role in facilitating a continuous mental transformation. As a test of generality, we next evaluated whether the continuity hypothesis holds in a different domain—mental arithmetic.

As noted in the 'Introduction' section, a large body of behavioural and neuroimaging research supports a contrast between algorithmic and memory retrieval processes involved in simple arithmetic.[33,35,46] For example, when participants solve addition problems with non-identical operands, RTs increase with the size of the operands (and their sum); in contrast, RTs remain flat when solving addition problems with identical operands.[43,60] Addition of non-identical operands has been posited to involve a continuous transformation of a spatialized mental representation (i.e. movement along a mental number line), whereas addition of identical operands is thought to involve rote memory retrieval from an overlearned look up table.[35,38,48]

The continuity hypothesis predicts that individuals with CD would be slower in performing calculations involving a mental number line. Specifically, the CD group would exhibit an elevated magnitude effect when adding non-identical operands (i.e. the RT difference between groups increases with the magnitude of the operands), mirroring the effect of rotation magnitude in Experiment 1. As a control task, we included arithmetic tasks primarily associated with memory retrieval from a look-up table.

Experiment 2a

Participants included 15 individuals with CD and 15 Control participants. We used a verification task in which the participants made a speeded response to indicate if addition equations were true or false (Fig. 4). The operands were between 3 and 8 and included all pairwise combinations and orders. Unlike Experiment 1, the continuous and non-continuous conditions were embedded within the same task: The continuous condition was composed of trials in which the two operands were non-identical (e.g. 5 + 8), problems presumed to be solved by 'moving' across a mental number line, while the non-continuous condition was composed of trials in which the two operands were identical (e.g. 6 + 6), problems presumed to be solved via rote memory retrieval.[43,57] The participants completed a single block that included 230 non-identical trials and 46 identical trials, with equations being true in half of the trials and false in the other half.

Reaction time effects

Overall RTs were slower in the non-identical condition compared to the identical condition [$F(1,7985) = 41.1$, $P < 0.001$, $\beta = 393.2$ (162.4, 624.0)], indicating that evaluating equations with non-identical operands required greater processing times than identical operands. There was an effect of Group, with the CD group showing slower RTs for both non-identical [CD = 2381 ± 1339 ms; Control = 1262 ± 757 ms; $t(1,26) = 4.1$, $P < 0.001$, $\beta = 1143$ (602, 1683)] and identical equations [CD = 1667 ± 833 ms; Control = 1054 ± 543 ms; $t(1,26) = 2.4$, $P = 0.02$, $\beta = 671$ (122.2, 1220.0)] relative to the control group. As noted in Experiment 1a, this large difference in overall RT may not arise solely from motor impairments associated with CD, an issue we address in the 'Discussion' section. RTs were neither impacted by Age [$F(1,26) = 0.2$, $P = 0.66$, $\beta = 5.2$ (–16.6, 27.0)] nor MoCA score [$F(1,26) = 0.16$, $P = 0.69$, $\beta = 25.1$ (–93.7, 143.8)].

Our main question centers on the increase in RT as a function of max operand. In line with previous work,[43,57] RTs increased with the max operand [$F(1,7985) = 123.9$, $P < 0.001$, $\beta = 83.3$ (63.1, 103.5)], requiring on average of ~83 ms more processing time per increment. However, there was an interaction between Problem Type and Max Operand [$F(1,7985) = 120.3$, $P < 0.001$, $\beta = -92.5$ (–130.5, –54.4)] (Fig. 5A and C). RT increased with Max Operand for the non-identical equations, consistent with the hypothesis that such equations require a time-demanding transformation along a number line. In contrast, RT was essentially flat for the identical equations. This latter effect has been observed in previous studies[43,57] and attributed to an invariant look-up time required for addition involving single-digit identical operands. RT slopes did not correlate with motor symptom severity in the CD group for either condition (identical: $R = -0.12$, $P = 0.67$; non-identical: $R = 0.19$, $P = 0.49$).

Turning to the group comparison, we observed a significant three-way interaction of Group, Problem Type and Max Operand [$F(1,7985) = 18.1$, $P < 0.001$, $\beta = -117.1$ (– 171.1, –63.1)]. For the non-identical equations, RT slopes were substantially larger in the CD group relative to controls [$t(7985) = 9.4$, $P < 0.001$, $\beta = 137.6$ (109.0, 166.2); Fig. 5A and B]. Individuals with CD required an additional 220.9 ± 10.3 ms per increment along the mental number line, whereas controls only required an additional 83.3 ± 10.3 ms per increment. In contrast, RT slopes were similar for the two groups [$t(7985) = -0.9$, $P = 0.82$, $\beta = 20.5$ (–25.2, 66.2); Fig. 5C and D], although, as noted above, the slope effect was not significant for equations involving identical operands [comparison to zero: Controls $t(7985) =$

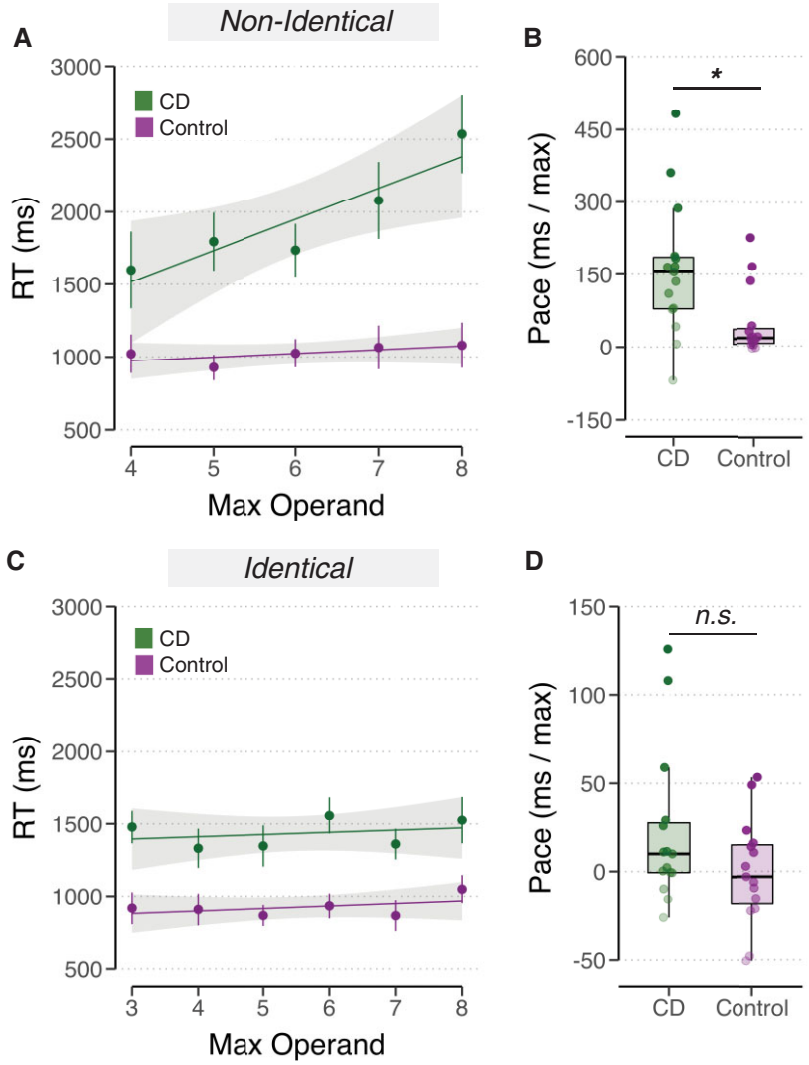

Figure 5 Reaction time analysis for Experiment 2a. Cerebellar degeneration associated with slower rate for addition problems putatively linked to continuous transformations along a number line, but not problems assumed to depend on memory retrieval. Median RT is plotted as a function of the maximum operand for equations with non-identical (**A**) and identical operands (**C**). Thin lines denote functions for each participant. The rate of adding non-identical (**C**) or identical operands (**D**) is estimated by the slope of each individual's RT function. Dots denote individuals. Mean regression lines are displayed in **A** and **C**. Shaded error bars denote 1 SEM. n.s. = not significant. *$P < 0.05$.

−0.6, $P = 0.58$, $\beta = -9.1$ (−41.4, 23.2); CD: $t(7985) = 0.7$, $P = 0.49$, $\beta = 11.4$ (−21.1, 43.8)]. Taken together, our results reveal that the CD group exhibited a specific impairment on the addition task associated with a continuous operation rather than a general deficit in mental arithmetic.

Accuracy effects

Accuracy was quite high (Supplementary Fig. 1C), with all of the individuals performing above 80% on both types of problems. Overall, accuracy was similar for the non-identical (94.8%, SD = 4.07) and identical equations [97.2%, SD = 3.40; $F(1,8517) = 0.6$, $P = 0.43$, $\beta = -0.02$ (−0.08, 0.05)]. Accuracy was neither impacted by Age [$F(1,26) = 1.0$, $P = 0.32$, $\beta = 0.00$ (0.00, 0.00)] nor MoCA score [$F(1,26) = 0.5$, $P = 0.43$, $\beta = 0.00$ (0.00, 0.00)]. Consistent with previous findings,[43] accuracy decreased as the max operand increased [$F(1,8517) = 8.1$, $P = 0.004$, $\beta = -0.01$ (−0.02, 0.00)].

The CD group performed worse than the control group on the non-identical problems [$t(28) = 2.3$, $P = -0.03$, $\beta = 0.03$ (−0.06, 0.00)], whereas the two performed equally well on the identical problems [$t(28) = 0.8$, $P = 0.41$, $\beta = -0.02$ (−0.06, 0.00)]. Arguing against any speed-accuracy trade-off concerns with the RT data, the three-way interaction of Group × Problem Type × Max Operand was not significant [$F(1,8517) = 0.1$, $P = 0.81$, $\beta = 0.00$ (−0.01, 0.02)] (Supplementary Fig. 1C). There were also no group differences in the slopes of the accuracy functions for the identical [$t(8516) = 0.12$, $P = 0.99$, $\beta = 0.0$ (0.0, 0.01)] and non-identical problems [$t(8516) = 0.65$, $P = 0.92$, $\beta = 0.00$ (0.00, 0.01)]. The rate of performance decline in the CD group was not related to motor symptom severity in either condition (non-identical: $R = 0.05$, $P = 0.87$; identical: $R = 0.13$, $P = 0.63$).

Experiment 2b

For the control condition in Experiment 2a, we selected equations involving identical numbers given prior evidence showing that these over-trained problems are generally solved via rote memory retrieval.[43,57] While this meets our objective for a non-continuous control task (i.e. one that involves a more discrete operation),

identical-operand addition is suboptimal in one important way: RTs do not increase as a function of the max operand. As such, the relatively constant time required to evaluate these equations (e.g. RT is essentially the same for 2 + 2 and 6 + 6) renders this rate measure difficult to interpret.

In Experiment 2b, we address this limitation by testing the participants on addition and multiplication problems, using the same set of operands for both tasks. The time required to perform and verify non-identical multiplication problems involving two single-digit numbers increases with the Max Operand, similar to that observed for non-identical addition problems.[45] However, the magnitude effect for multiplication has been associated with differences in the time required to access a look-up table, an effect related to the associative strength between operands and their product, likely mediated by experience and repetition.[42,61] Addition and multiplication have also been dissociated in the brain, with the former tapping into areas linked to spatial manipulation and the latter relying on areas linked to verbal memory.[35,48] By using multiplication as a non-continuous control task, we expected to obtain meaningful rate measures for both tasks.

The continuity hypothesis predicts that that the CD group would show elevated RT slopes only for the addition problems, replicating Experiment 2a, but not for the multiplication problems. An added feature of comparing addition and multiplication is that, *a priori*, we assumed that multiplying two single-digit numbers is more difficult than adding the same numbers. An impairment associated with CD in this experiment is thus predicted for the easier task. Fifteen individuals with CD and 15 Controls were tested in Experiment 2b (of these, eight individuals with CD and seven Controls also were tested in Experiment 1b).

Reaction time effects

We first consider the addition and multiplication equations with non-identical operands. RTs were not significantly different for the addition and multiplication problems [$F(1,12\,983) = 28.5$, $P = 0.18$, $\beta = 143.8$ (−65.4, 352.9)]. Similar to Experiment 2a, the CD group was slower than the control group on both tasks [Addition: CD 1990 ± 897 ms; Control 1305 ± 565 ms; $t(26) = 2.8$, $P = 0.01$, $\beta = 520$ (155.4, 884.6); Multiplication: CD 2346 ± 1269 ms; Control 1505 ± 818 ms, $t(26) = 3.6$, $P = 0.001$, $\beta = 672$ (307.4, 1036.6)]. RTs were not impacted by Age [$F(1,26) = 2.8$, $P = 0.10$, $\beta = 10.2$ (−1.3, 21.8)] or MoCA scores [$F(1,26) = 3.3$, $P = 0.08$, $\beta = -76.0$ (−155.4, 3.4)].

RTs increased with the Max Operand [$F(1,12\,983) = 224.2$, $P < 0.001$, $\beta = 62.9$ (43.7, 82.1)] and, unlike Experiment 2a, this effect was observed with both types of problem; the Problem Type × Max Operand interaction was not significant [$F(1,12\,983) = 2.7$, $P = 0.10$, $\beta = 8.2$ (−19.0, 35.4)]. The RT slopes in the CD group did not correlate with SARA scores in either task (addition: $R = -0.27$, $P = 0.32$; multiplication: $R = -0.29$, $P = 0.30$).

We again observed a significant three-way interaction of Group, Problem Type and Max Operand [$F(1,12\,983) = 6.1$, $P = 0.01$, $\beta = -48.5$ (−87.0, −10.0)]. Consistent with the continuity hypothesis, the computation rate for addition was selectively elevated in the CD group compared to the Controls [$t(12\,983) = 2.7$, $P = 0.03$, $\beta = 37.3$ (10.2, 64.3); Fig. 6A]. The CD group required an additional 100.2 ± 9.7 ms for each integer increment on the mental number line; the comparable value for the Control group was only 63.9 ± 9.8 ms/integer. In contrast, the computation rate for multiplication was similar between groups [$t(12\,983) = 0.8$, $P = 0.85$, $\beta = 11.3$ (−16.2, 38.7); Fig. 6A], a null effect that persisted even when the three CD participants exhibiting negative slopes in the multiplication RT function were removed [$t(11\,726) = -1.7$, $P = 0.33$, $\beta = -23.5$ (−50.8, 3.7); Fig. 6D]. The mean rate values were 59.9 ± 10.0 ms/integer for the CD group and 71.1 ± 9.8 ms/integer for the Control group. Taken together, the dissociation is consistent with the continuity hypothesis, namely that cerebellar degeneration impacts a continuous operation performed along a mental number line for simple addition but does not affect rote memory retrieval required for single-digit multiplication.

In a *post hoc* analysis, we performed a between-experiment comparison of the addition problems involving non-identical operands. This analysis showed that RT slopes in this condition were similar in Experiment 2a for the CD group [$t(28) = 1.8$, $P = 0.08$ (−8.4, 149.1)] and Control group [$t(28) = 0.4$, $P = 0.67$ (−35.0, 53.9)], signalling a successful replication in this critical condition.

Accuracy effects

Accuracy tended to be higher on the addition problems (95.5%, SD = 0.03) compared with the multiplication problems (91.7%, SD = 5.52), although the difference was not significant [$F(1,14\,364) = 1.7$, $P = 0.08$, $\beta = 0.06$ (−0.01, 0.12)] (Supplementary Fig. 1D), perhaps due to a ceiling effect for some participants. Accuracy did not vary with Age [$F(1,26) = 2.8$, $P = 0.11$, $\beta = 0.00$ (0.00, 0.00)] and was unrelated to MoCA scores [$F(1,26) = 1.4$, $P = 0.24$, $\beta = 0.00$ (−0.01, 0.00)]. Error rates increased as the Max Operand increased [$F(1,14\,364) = 98.0$, $P < 0.001$, $\beta = -0.01$ (−0.01, 0.00)], an effect observed with both types of equations (Supplementary Fig. 1D).

The two groups were similar in terms of accuracy for the addition problems [$t(29) = 0.2$, $P = 0.86$, $\beta = 0.00$ (−0.03, 0.04)]. In contrast, accuracy was worse in the CD group for the multiplication problems [$t(29) = 3.8$, $P = 0.001$, $\beta = -0.07$ (−0.10, −0.03)], raising the possibility that the null group effect in the RT slope data in multiplication might be due to a speed-accuracy trade-off. Indeed, there was a significant three-way interaction in the accuracy data [$F(1,14\,364) = 7.2$, $P = 0.007$, $\beta = -0.02$ (−0.03, 0.00)]: Accuracy in multiplication decreased faster as a function of Max Operand in the CD group compared with the Control group [$t(14\,364 = 3.2$, $P = 0.008$, $\beta = -0.01$ (−0.02, −0.01)], whereas the two groups had a similar rate of decrement for the addition problems [$t(14\,364) = -0.6$, $P = 0.93$, $\beta = 0.00$ (−0.01, 0.01)]. This slope values for the CD group did not vary with clinical motor severity in either task (addition: $R = 0.10$, $P = 0.70$; multiplication: $R = 0.05$, $P = 0.86$).

To evaluate whether a potential group difference in RT slopes in multiplication were masked by the group difference in accuracy, we closely examined the relationship between the slope values for the accuracy and RT data: If participants were trading speed for accuracy, then participants who show a larger slope for the accuracy function should show a smaller slope for the RT function and vice versa. We neither observed this negative correlation in the control group [multiplication: $R = -0.11$ (−0.59, 0.43), $P = 0.70$; addition: $R = 0.07$ (−0.46, 0.56), $P = 0.79$], nor in the CD group [multiplication: $R = 0.27$ (−0.28, 0.69), $P = 0.32$; addition: $R = 0.0$ (−0.51, 0.52), $P = 0.98$]. In summary, the CD group's impairment in RTs does not originate from a speed-accuracy trade-off. Instead, the slope difference in accuracy between the two groups on the multiplication task may be attributed to a ceiling effect that depresses the slope for the control group.

Comparison of problems with identical operands

While our main hypothesis in Experiment 2b centered on the comparison of addition and multiplication equations with non-identical operands, the addition and multiplication conditions included equations with identical operands. RT slopes were not significantly different between groups for addition with identical operands [$t(28) = 0.0$, $P = 0.97$ (−16.1, 16.6)] and multiplication with identical operands [$t(28) = -1.0$, $P = 0.32$ (−69.7, 23.5)]. These results

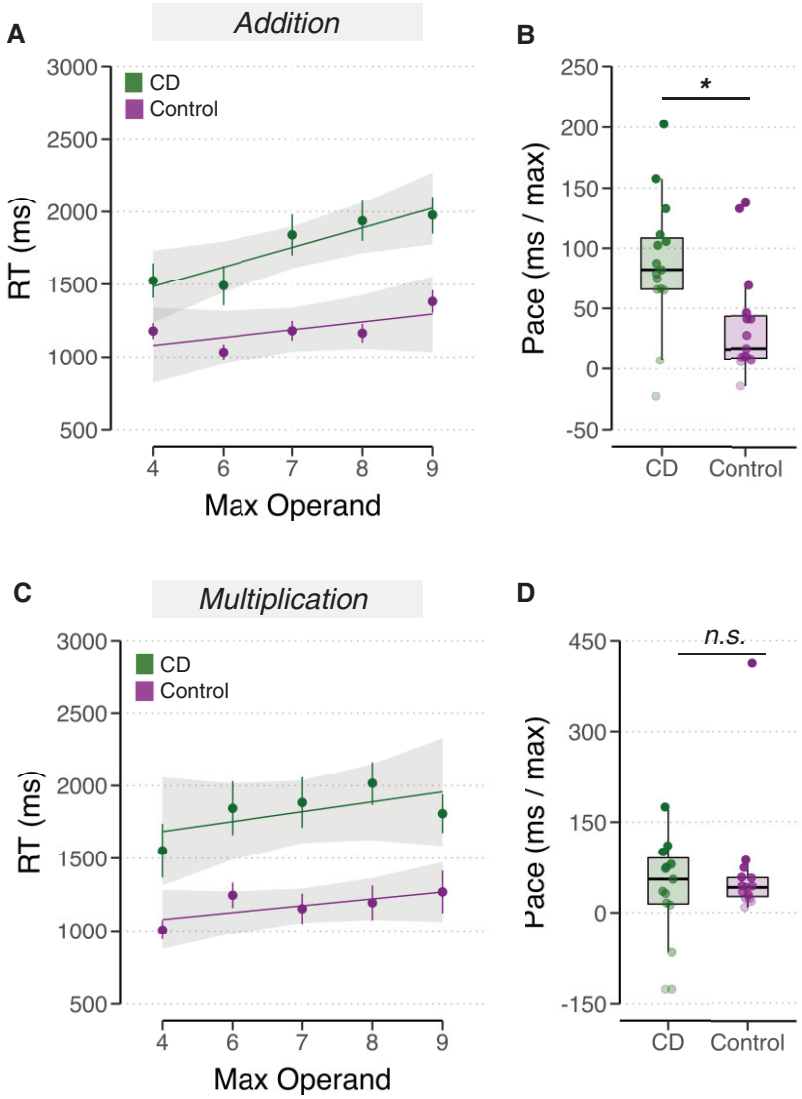

Figure 6 Reaction time analysis for Experient 2b. Replication of selective impairment in CD group on arithmetic problems associated with continuous transformations along a number line. Median RT is plotted as a function of the maximum operand for addition (**A**) and multiplication (**C**), using equations with non-identical operands only. Thin lines denote functions for each participant. The rate for addition (**C**) or multiplication (**D**), is estimated by the slope of each individual's RT function. Dots denote individuals. Mean regression lines are displayed in **A** and **C**. Shaded error bars denote 1 SEM. n.s. = not significant. $^{*}P < 0.05$.

provide further evidence that cerebellar degeneration does not impact the speed of performing arithmetic operations assumed to be dependent on memory retrieval.

Accuracy was very high in both tasks (identical addition: CD: 96.94%, SD = 2.71%; Control: 98.89%, SD = 1.55%; identical multiplication: CD: 88.75%, SD = 9.12%; Control: 94.44%, SD = 5.66%), and again, the rate at which performance worsened with increasing operands was similar across groups [identical addition: $t(28) = 1.4$, $P = 0.18$ (0.00, 0.02); $t(28) = -0.9$, $P = 0.35$ (−0.03, 0.01)].

Analyses limited to CD subtypes not typically associated with profound extracerebellar involvement

Our sample of individuals with CD was rather heterogeneous and included individuals with subtypes known to involve extracerebellar pathology (e.g. SCA1, SCA2, SCA3, SCA5, SCA8, SCA15, SCA28 and AOA2).[50,51] This raises the question of whether the observed pattern of deficits originates from degeneration of the cerebellum or pathology outside the cerebellum (or some combination of the two). To address this question, we performed a series of secondary analyses on the data from the 10 individuals with SCA6 and 22 individuals with SAOA, subtypes in which extracerebellar pathology is thought to be relatively limited. Even with the limited samples in these analyses, the results were consistent with predictions derived from the continuity hypothesis. In Experiment 1a, this led to the exclusion of five individuals with CD (Supplementary Table 1). The CD group showed a slower rate of mental rotation compared to the full sample of control participants [$t(5411) = 4.3$, $P = 0.001$, $\beta = 0.9$ (0.5, 1.3)] but exhibited a similar rate of memory search [$t(5411) = 2.2$, $P = 0.12$, $\beta = -23.2$ (−43.8, 2.7)]. The same interaction was observed in Experiment 1b where only one individual with CD was excluded [mental rotation: $t(7815) = 3.0$, $P = 0.01$, $\beta = 0.7$ (0.3, 1.2); memory search: $t(7815) = 0.4$, $P = 0.98$, $\beta = 6.0$ (−23.2, 35.1)].

Experiment 2a was most impacted by this selection criterion with nine of the 15 participants excluded due to a diagnosis

associated with potential extracerebellar pathology. Nonetheless, the remaining six individuals in the CD group exhibited elevated RT slopes on the non-identical addition problems [$t(6154) = 6.2$, $P < 0.001$, $\beta = 97.8$ (67.0, 128.6)] but normal RT slopes on the identical addition problems [$t(6154) = -0.5$, $P = 0.95$, $\beta = 13.7$ (−35.5, 62.9)]. The key results were also unchanged in Experiment 2b after excluding two patients: The CD group exhibited elevated RT slopes for the addition problems [$t(12\,126) = 2.5$, $P = 0.05$, $\beta = 37.1$ (8.5, 65.7)] but normal RT slopes for the multiplication problems [$t(12\,126) = 0.8$, $P = 0.84$, $\beta = -12.2$ (−41.2, 16.8)].

We recognize the limitations with these secondary analyses, both in terms of the assumptions underlying the selection criterion and consequent smaller sample sizes. Even in the so-called 'pure cerebellar syndromes' (SCA6 and SAOA) there may be extracerebellar pathology, especially in advanced cases.[62,63] Moreover, individuals in our SCA6 group (tested only in Experiments 1b and 2b) were all related, which may introduce a family effect. Nonetheless, these analyses are consistent with the hypothesis that the impairment in efficiently executing continuous mental transformations is related to pathology of the cerebellum. Interestingly, the clinical measures of ataxia and mental status failed to reveal systematic relationships with diagnostic category or task performance: The SARA and MoCA scores were not associated with the deficits observed in any of the four experiments. The null results for the MoCA are especially noteworthy given that some of the CD participants scored in the range associated with mild cognitive impairment, reinforcing the notion that deficits in continuous mental transformations appear to be linked to cerebellar pathology rather than general cognitive decline.

Discussion

A large body of experimental and theoretical work has yielded detailed models of how the cerebellum supports sensorimotor learning and motor control. Although the involvement of the human cerebellum in cognition has been highlighted across many studies since the seminal conjecture of Leiner, Leiner and Dow,[1] our understanding of its functional role in non-motor behaviours remains limited. The diverse patterns of task-related activity observed in neuroimaging studies of the human cerebellum might imply a heterogeneous role for the cerebellum in cognition.[10,64] Alternatively, the homogeneous anatomy and physiology of the cerebellum has inspired the idea that the cerebellum may invoke a common computation across diverse task domains,[2,18] a so-called 'universal cerebellar transform' (UCT). From this view-point, the diverse cerebellar activation patterns observed in the neuroimaging literature[10,64] are seen as reflecting the diversity of inputs to the human cerebellum,[65] with a UCT being applied to these inputs to support a range of behaviours. While recognizing that homogenous structure and physiology need not imply homogenous function, the UCT concept has helped generate testable computational hypotheses.[19,21,24,66,67]

A continuity constraint on cerebellar involvement in cognition

In the present study, we build on the UCT concept, seeking to identify constraints on the type of cognitive operations that are likely to rely on the cerebellum. Our theorizing draws on the distinction between continuous and discrete mental operations, a dimension that has proven useful in characterizing mental transformations.[26] The dynamics required to efficiently manipulate a body are inherently continuous, and the hallmark of cerebellar ataxia is a loss of fluidity in the movements required to transition smoothly from one state to another.[20,24] In a similar manner, we propose that a continuity constraint may generalize beyond the motor domain, positing that the cerebellum facilitates processing in tasks that involve a continuous transformation of mental representations. This continuity hypothesis offers one novel constraint on the cerebellum and cognition: It specifies conditions under which the cerebellum would facilitate the efficient, coordinated operation of mental operations and, as a corollary, conditions in which the cerebellum might be expected to have a reduced, or even negligible role.

We evaluated the continuity hypothesis in two disparate domains. In the domain of visual cognition, we compared two classes of tasks: A mental rotation task known to rely on a continuous transformation,[27,28,30] and two variants of a visual memory search task presumed to require retrieval of discrete representations in working memory. RT slopes, taken as a proxy for the core mental operation required in each task, were elevated in the cerebellar degeneration group for mental rotation but neither for visual (Experiment 1a) nor visuospatial (Experiment 1b) working memory search. This dissociation was observed in two independent samples of individuals with degenerative CDs and matched controls. Moreover, this dissociation was robust to outlier and subgroup analyses.

We also evaluated the continuity hypothesis in a different cognitive domain, mathematical cognition. A distinction between continuous and discrete operations has also been articulated in this domain.[33,38,40,43] Inspired by this literature, we selected simple addition as a continuous operation given the evidence that this operation primarily entails a translation across a mental number line.[34,38,40,68] For the discrete operation, we selected addition of identical single-digit integers or multiplication, operations hypothesized to rely on rote retrieval from memory. Consistent with our predictions, individuals with CD were selectively impaired on the simple addition problems, manifest as a steeper increase in RT with operand magnitude compared to that observed in controls. In contrast, RT slopes were similar between CD and control groups on the two arithmetic tasks that primarily depend on memory retrieval. As in Experiment 1, this dissociation was observed in two independent samples of participants.

We recognize that there are substantial differences in these continuous and discrete tasks. This is especially true in Experiment 1, where performance on mental rotation was compared with performance on two visual working memory tasks. There are alternative ways to interpret the computational differences between the continuous and discrete tasks; of note, in our experiments the latter all involve some form of memory retrieval. Thus, the current results would also be consistent with the idea that the cerebellum is not essential for the retrieval of information from memory but becomes essential when the retrieved information must be dynamically manipulated in some manner. Future research is required to test this related hypothesis, or perhaps examine how these two framings of our results can be synthesized.

Study limitations

The continuity hypothesis cannot account for all the behavioural differences observed between the CD and control groups. First, the CD group consistently responded more slowly in all tasks and conditions, not just those that required a continuous mental transformation. The magnitude of this group difference is unlikely to be

fully attributable to the motor control problems associated with ataxia. For example, baseline RT differences between the CD and control groups were around ~320 ms across the two mental rotation tasks and ~470 ms across the two working memory search tasks, values considerably larger than those observed in tasks involving relatively simple perceptual discriminations.[69] We assume that some of the increase in RT is related to global deficits associated with degenerative disorders such as CD (e.g. general resource allocation issues, fatigue, etc.).

Second, although not dependent on set size, the CD group had worse accuracy than the control group on both memory search tasks. These results again suggest that in addition to a specific impairment in performing the continuous mental operations (i.e. the slope effects), the CD participants may also have impairments that influence performance in a more generic manner. Speculatively, if the continuous coordination of top-down attention leverages the cerebellum in a manner consistent with our hypothesis, CD could lead to global deficits across a range of demanding cognitive tasks. A more global deficit of this type could also help reconcile why we observed no impairment in the CD group on measures of search rate through working memory, yet the clinical literature shows that patients with CD tend to exhibit modest impairments on neuropsychological assessments of 'frontal function', including tests of working memory.[70,71] These clinical instruments tend to provide a more global picture of competence rather than home in on specific mental operations.

We also observed group differences in accuracy in the multiplication task (Experiment 2b), a deficit that became more pronounced as the operand increased. While subsequent analyses ruled out a speed accuracy account of the null group effect for the RT data in this condition, we do not have a ready account of this accuracy effect. We note that in designing these experiments, we deliberately avoided the use of accuracy as a primary dependent variable because this measure is sensitive to many cognitive processes: Poor accuracy can stem from attentional lapses, encoding errors, motor errors, response biases, fatigue, etc., and all of these would be expected to compound with problem difficulty.[72] For these reasons, we selected tasks that allowed us to focus on parametric changes in RT (limited to correct trials), as this type of dependent variable provides a proxy of more task-specific cognitive operations (e.g. the rate of mental rotation, movement along a mental number line, etc.). Strikingly, across four separate experiments in two disparate cognitive domains, we observed a consistent three-way interaction in the RT data, showing a dissociation between the CD and Control groups on the continuous and discrete tasks.

Our study had additional limitations related to the individuals recruited for the CD group. In particular, our study population was rather heterogeneous, both in terms of diagnosis and genetic aetiology (Supplementary Table 1). Moreover, the sample included individuals with diagnoses in which extracerebellar pathology is likely. This raises the possibility that the observed continuity impairment is related to extracerebellar pathology rather than cerebellar pathology. We find this hypothesis to lack parsimony on both theoretical and empirical grounds. Theoretically, the putative extracerebellar region(s) would have to be sensitive to the specific constraint we hypothesized to be cerebellar-dependent, one derived *a priori* based on computational models of cerebellar function in sensorimotor control and learning. It is hard to envision how current computational models of the extracerebellar regions impacted in a subset of our CD sample (e.g. pons, olive, and/or basal ganglia) would result in the selective pattern of impairment observed across the experiments. Empirically, the main results were replicated in secondary analyses of all four experiments in which we excluded individuals from the subgroups most strongly associated with extracerebellar pathology. This pattern strongly suggests that it is degeneration of the cerebellum which is the common factor driving the results.

Nonetheless, future research involving more homogeneous clinical groups, coupled with structural and functional MRI data, could paint a more detailed picture of the networks engaged in operations required for continuous mental transformations. In this study, we did not have sufficient radiological data to rigorously examine structure-behaviour relationships (e.g. voxel-based morphometry). This would be useful to not only evaluate the relevance of extracerebellar pathology but could also provide insight into subregions of the cerebellum that are associated with performance on our experimental tasks. Similarly, functional connectivity data could be used to examine network connectivity patterns between subregions of the cerebellum and the neocortex in our tasks.

Lastly, another limitation involves the liberal screening cutoff we adopted for the MoCA exam, one that resulted in a sample that included individuals who would be classified as having mild cognitive impairment (MCI). *A priori*, a liberal inclusion criterion seemed appropriate given our interest in the contributions of the cerebellum to cognition—to exclude those with mild cognitive impairment could induce a bias to conclude that CD does not impact cognition. More importantly, we do not think this criterion is problematic in terms of the interpretation of the results. First, we matched MoCA scores across groups in three of the four experiments. Second, we included MoCA scores as a covariate in all key analyses. Third, one would expect that MCI would produce global cognitive deficits on all the experimental tasks, rather than a selective deficit on tasks involving continuous transformation. Lastly, we also note that the MoCA may overestimate MCI in individuals with CD given the motor requirements for several of the test items. In future work, we will leverage the scale specifically designed to evaluate cerebellar cognitive affective syndrome[71] to obtain a more complete neuropsychological profile.

Relating the continuity hypothesis to previous work on the role of the cerebellum in cognition

The impairment we observed on mental rotation converges with prior work showing that deficits in visuospatial reasoning in individuals with cerebellar pathology (lesion or atrophy) are most pronounced on tasks requiring some sort of mental transformation.[73] However, the null results on the two visual working memory tasks in Experiment 1 would appear, at first blush, to be at odds with the rather substantial neuroimaging literature pointing to the involvement of the cerebellum in working memory.[10,31,74,75] We note that the picture is less clear on the neuropsychological front, with some studies reporting impairments in individuals with cerebellar pathology on working memory tasks,[15,76] whereas others report null effects on these tasks.[12,77] Importantly, the designs in prior studies have tended to focus on main effects; for example, is activation higher on the working memory task relative to some baseline control, or do patients have reduced working memory span? As shown in the current work, although the CD group was slower on all tasks, the comparison of rate-dependent measures was essential for revealing the dissociation between the continuous and discrete tasks. A similar approach could be adopted for a neuroimaging study. For example, we would predict that the magnitude of the brain-oxygen level dependent response in the

cerebellum could be correlated with the rate of mental rotation, but not the rate at which items are retrieved from working memory.

The continuity hypothesis can also be used to re-examine work in other task domains. To give one example from our own work on temporal cognition, we have shown that individuals with CD are selectively impaired in using an interval-based representation for the production of periodic movement[78] and temporal orienting.[69] In such tasks, performance requires reference to a continuously updated internal representation of time. In contrast, these individuals were unimpaired when temporal control can be achieved by reference to a constant control parameter (e.g. angular velocity in tapping, entrainment to an exogenous referent signal in temporal orienting). Although admittedly *post hoc*, we speculate that the continuity constraint might also offer new insight into the functional domain of the cerebellum in timing, emphasizing its importance in the continuous updating of an internal state variable.

Considered more broadly, an important benefit of the continuity hypothesis is that it offers a specific, falsifiable model of one potential way in which the cerebellum contributes to cognition. In this sense, the continuity hypothesis departs from previous conjectures that are more descriptive. For instance, while the concept of 'dysmetria of thought' is an apt summary description of the clinical profile of the cognitive deficits associated with cerebellar pathology,[16] it is unclear how this hypothesis can be operationalized, especially when considering a broad range of cognitive domains. In contrast, the continuity hypothesis can be readily tested in other domains. For example, some models of top-down visual attention posit a covert 'spotlight' that continuously traverses the visual scene.[79,80] Speculatively, we would predict that individuals with CD could be impaired at smoothly guiding a putative attentional spotlight through space, just as they are with guiding their eyes and limbs.

Our hypothesis also departs from previous conjectures that are more general in nature. For example, while 'prediction' has served as an important concept for understanding the role of the cerebellum in motor control,[24] it remains unclear how this concept should be applied to more cognitive domains, and more specifically, how the cerebellum's predictive capacity might differ from predictive operations performed by the rest of the brain.[25] In motor control, the region's predictive capacity is thought to support the transformations needed to guide a limb in a continuous manner from position A to position B. Mental rotation could be posed as a similar kind of continuous state transition problem; e.g. how do I rotate this mental object from orientation A to orientation B? Traversal of a mental number line could require an analogous state transition from one point in mental space to another (e.g. from left to right). From this view-point, our hypothesis shifts the focus away from a singular emphasis on prediction in general and more towards consideration of the constraints on cerebellar-dependent predictions. In a similar vein, the continuity constraint is relevant in thinking about the role of the cerebellum in supporting internal models.[24] Simulation via an internal model provides a means to link together states, sensorimotor or otherwise, that are closely related in time.[81] As with prediction, we expect that the continuity constraint specifies the boundary conditions of the cerebellum in supporting such simulations.

In summary, the continuity hypothesis provides a novel perspective on the contributions of the cerebellum to cognition, one that seeks to offer some computational specificity as to how the cerebellum facilitates mental coordination and prediction. More generally, postulating constraints on cerebellar computation is essential for advancing our understanding of how this subcortical structure interacts with the rest of the brain to support our motor and mental competences. Whether the concept of a universal cerebellar transform has utility will require evaluating the relevance of such constraints across a broad range of cognitive domains.

Acknowledgements

We thank Sam Wang and Dagmar Timmann for thoughtful discussions. The order of co-first authors was determined by alphabetical order.

Funding

R.B.I. is funded by the NIH (grants NS116883 and NS105839). J.A.T. is funded by the NIH (grant NS084948). S.D.M. was funded by NIH fellowship F32 MH119797. J.S.T. is funded by the PODS II scholarship from the Foundation for Physical Therapy Research and by NIH (NRSA F31: 1F31NS120448-01A1).

Competing interests

The authors report no competing interests.

Supplementary material

Supplementary material is available at *Brain* online.

References

1. Leiner HC, Leiner AL, Dow RS. Does the cerebellum contribute to mental skills? *Behav Neurosci*. 1986;100(4):443–454.
2. Schmahmann JD, Guell X, Stoodley CJ, Halko MA. The theory and neuroscience of cerebellar cognition. *Annu Rev Neurosci*. 2019;42:337–364.
3. Buckner RL, Krienen FM, Castellanos A, Diaz JC, Yeo BTT. The organization of the human cerebellum estimated by intrinsic functional connectivity. *J Neurophysiol*. 2011;106(5):2322–2345.
4. Middleton FA, Strick PL. Cerebellar projections to the prefrontal cortex of the primate. *J Neurosci*. 2001;21(2):700–712.
5. Strick PL, Dum RP, Fiez JA. Cerebellum and nonmotor function. *Annu Rev Neurosci*. 2009;32(1):413–434.
6. Badura A, Verpeut JL, Metzger JW, *et al*. Normal cognitive and social development require posterior cerebellar activity. *eLife*. 2018;7:e36401.
7. Carta I, Chen CH, Schott AL, Dorizan S, Khodakhah K. Cerebellar modulation of the reward circuitry and social behavior. *Science*. 2019;363(6424):eaav0581.
8. Deverett B, Koay SA, Oostland M, Wang SSH. Cerebellar involvement in an evidence-accumulation decision-making task. *eLife*. 2018;7:e36781.
9. Buckner RL. The cerebellum and cognitive function: 25 years of insight from anatomy and neuroimaging. *Neuron*. 2013;80(3):807–815.
10. King M, Hernandez-Castillo CR, Poldrack RA, Ivry RB, Diedrichsen J. Functional boundaries in the human cerebellum revealed by a multi-domain task battery. *Nat Neurosci*. 2019;22(8):1371–1378.
11. Moberget T, Gullesen EH, Andersson S, Ivry RB, Endestad T. Generalized role for the cerebellum in encoding internal models: Evidence from semantic processing. *J Neurosci*. 2014;34(8):2871–2878.

12. Alexander MP, Gillingham S, Schweizer T, Stuss DT. Cognitive impairments due to focal cerebellar injuries in adults. *Cortex*. 2012;48(8):980–990.
13. Malm J, Kristensen B, Karlsson T, Carlberg B, Fagerlund M, Olsson T. Cognitive impairment in young adults with ipfratentorial infarcts. *Neurology*. 1998;51(2):433–440.
14. Cooper FE, Grube M, Von Kriegstein K, *et al*. Distinct critical cerebellar subregions for components of verbal working memory. *Neuropsychologia*. 2012;50(1):189–197.
15. Ravizza SM, McCormick CA, Schlerf JE, Justus T, Ivry RB, Fiez JA. Cerebellar damage produces selective deficits in verbal working memory. *Brain*. 2006;129(2):306–320.
16. Schmahmann JD, Sherman JC. The cerebellar cognitive affective syndrome. *Brain*. 1998;121:561–579.
17. Schmahmann JD. An emerging concept: The cerebellar contribution to higher function. *Arch Neurol*. 1991;48(11):1178.
18. Diedrichsen J, King M, Hernandez-Castillo C, Sereno M, Ivry RB. Universal transform or multiple functionality? understanding the contribution of the human cerebellum across task domains. *Neuron*. 2019;102(5):918–928.
19. Ito M. Control of mental activities by internal models in the cerebellum. *Nat Rev Neurosci*. 2008;9(4):304–313.
20. Miall RC, Christensen LOD, Cain O, Stanley J. Disruption of state estimation in the human lateral cerebellum. *PLOS Biol*. 2007;5-(11):e316.
21. Ivry RB, Keele SW. Timing functions of the cerebellum. *J Cogn Neurosci*. 1989;1(2):136–152.
22. Smith MC. CS-US interval and US intensity in classical conditioning of the rabbit's nictitating membrane response. *J Comp Physiol Psychol*. 1968;66(3, Pt.1):679–687.
23. Synofzik M, Lindner A, Thier P. The cerebellum updates predictions about the visual consequences of one's behavior. *Curr Biol*. 2008;18(11):814–818.
24. Wolpert DM, Miall RC, Kawato M. Internal models in the cerebellum. *Trends Cogn Sci*. 1998;2(9):338–347.
25. Friston K. Does predictive coding have a future? *Nat Neurosci*. 2018;21(8):1019–1021.
26. Miller J. Discrete and continuous models of human information processing: Theoretical distinctions and empirical results. *Acta Psychol*. 1988;67(3):191–257.
27. Kosslyn S. The medium and the message in mental imagery: A theory. *Psychol Rev*. 1981;88(1):46–66.
28. Shepard RN, Metzler J. Mental rotation of three-dimensional objects. *Science*. 1971;171(3972):701–703.
29. Georgopoulos AP, Lurito JT, Petrides M, Schwartz AB, Massey JT. Mental rotation of the neuronal population vector. *Science*. 1989; 243(4888):234–236.
30. Georgopoulos AP, Pellizzer G. The mental and the neural: Psychological and neural studies of mental rotation and memory scanning. *Neuropsychologia*. 1995;33(11):1531–1547.
31. Stoodley CJ. The cerebellum and cognition: Evidence from functional imaging studies. *Cerebellum*. 2012;11(2):352–365.
32. Sternberg S. High-speed scanning in human memory. *Science*. 1966;153(3736):652–654.
33. Dehaene S. Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Science*. 1999;284(5416):970–974.
34. Dehaene S, Bossini S, Giraux P. The mental representation of parity and number magnitude. *J Exp Psychol: Gen*. 1993;122(3):371–396.
35. Dehaene S, Cohen L. Cerebral pathways for calculation: Double dissociation between rote verbal and quantitative knowledge of arithmetic. *Cortex*. 1997;33(2):219–250.
36. Hubbard EM, Piazza M, Pinel P, Dehaene S. Interactions between number and space in parietal cortex. *Nat Rev Neurosci*. 2005;6(6): 435–448.
37. Mathieu R, Gourjon A, Couderc A, Thevenot C, Prado J. Running the number line: Rapid shifts of attention in single-digit arithmetic. *Cognition*. 2016;146:229–239.
38. Pinheiro-Chagas P, Dotan D, Piazza M, Dehaene S. Finger tracking reveals the covert stages of mental arithmetic. *Open Mind*. 2017;1(1):30–41.
39. Verguts T, Van Opstal F. Dissociation of the distance effect and size effect in one-digit numbers. *Psychono Bull Rev*. 2005;12(5): 925–930.
40. McCrink K, Dehaene S, Dehaene-Lambertz G. Moving along the number line: Operational momentum in nonsymbolic arithmetic. *Percept Psychophys*. 2007;69(8):1324–1333.
41. McCrink K, Wynn K. Operational momentum in large-number addition and subtraction by 9-month-olds. *J Exp Child Psychol*. 2009;103(4):400–408.
42. Campbell JID, Graham DJ. Mental multiplication skill: Structure, process, and acquisition. *Can J Psychol*. 1985;39(2): 338–366.
43. Groen GJ, Parkman JM. A chronometric analysis of simple addition. *Psychol Rev*. 1972;79(4):329–343.
44. Campbell JID, Xue Q. Cognitive arithmetic across cultures. *J Exp Psychol: Gen*. 2001;130:299–315.
45. LeFevre JA, Bisanz J, Daley K, Buffone L, Greenham S, Sadesky G. Multiple routes to solution of single-digit multiplication problems. *J Exp Psychol: Gen*. 1996;125(3):284–306.
46. Fayol M, Thevenot C. The use of procedural knowledge in simple addition and subtraction problems. *Cognition*. 2012;123(3): 392–403.
47. Galfano G, Rusconi E, Umiltà C. Automatic activation of multiplication facts: Evidence from the nodes adjacent to the product. *Q J Exp Psychol A*. 2003;56(1):31–61.
48. Zhou X, Chen C, Zang Y, *et al*. Dissociated brain organization for single-digit addition and multiplication. *NeuroImage*. 2007;35(2): 871–880.
49. Schmitz-Hubsch T, du Montcel ST, Baliko L, *et al*. Scale for the assessment and rating of ataxia: Development of a new clinical scale. *Neurology*. 2006;66(11):1717–1720.
50. Klockgether T, Mariotti C, Paulson HL. Spinocerebellar ataxia. *Nat Rev Dis Primers*. 2019;5(1):1–21.
51. Klockgether T, Paulson H. Milestones in ataxia. *Mov Disorder*. 2011;26(6):1134–1141.
52. Hernandez-Castillo CR, King M, Diedrichsen J, Fernandez-Ruiz J. Unique degeneration signatures in the cerebellar cortex for spinocerebellar ataxias 2, 3, and 7. *NeuroImage Clin*. 2018;20:931–938.
53. Brainard DH. The psychophysics toolbox. *Spat Vis*. 1997;10:433–436.
54. Young JM, Palef SR, Logan GD. The role of mental rotation in letter processing by children and adults. *Can J Exp Psychol*. 1980; 34(3):265–269.
55. Sklar AY, Levy N, Goldstein A, Mandel R, Maril A, Hassin RR. Reading and doing arithmetic nonconsciously. *Proc Natl Acad Sci*. 2012;109(48):19614–19619.
56. Schielzeth H, Dingemanse NJ, Nakagawa S, *et al*. Robustness of linear mixed-effects models to violations of distributional assumptions. *Methods Ecol Evol*. 2020;11(9):1141–1152.
57. Barrouillet P, Thevenot C. On the problem-size effect in small additions: Can we really discard any counting-based account? *Cognition*. 2013;128(1):35–44.
58. Uittenhove K, Thevenot C, Barrouillet P. Fast automated counting procedures in addition problem solving: When are they used and why are they mistaken for retrieval? *Cognition*. 2016;146: 289–303.
59. Leggio MG, Molinari M, Neri P, Graziano A, Mandolesi L, Petrosini L. Representation of actions in rats: The role of

cerebellum in learning spatial performances by observation. *Proc Natl Acad Sci USA*. 2000;97(5):2320–2325.
60. Restle F. Speed of adding and comparing numbers. *J Exp Psychol*. 1970;83(2, Pt.1):274–278.
61. Miller K, Perlmutter M, Keating D. Cognitive arithmetic: Comparison of operations. *J Exp Psychol Learn Mem Cogn*. 1984;10(1):46–60.
62. Schöls L, Krüger R, Amoiridis G, Przuntek H, Epplen JT, Riess O. Spinocerebellar ataxia type 6: Genotype and phenotype in German kindreds. *J Neurol Neurosurg Psychiatry*. 1998;64(1):67–73.
63. Harding AE. The clinical features and classification of the late onset autosomal dominant cerebellar ataxias. A study of 11 families, including descendants of the 'the Drew family of Walworth'. *Brain*. 1982;105(Pt 1):1–28.
64. Keren-Happuch E, Chen SHA, Ho MHR, Desmond JE. A meta-analysis of cerebellar contributions to higher cognition from PET and fMRI studies: A meta-analysis of cerebellar contributions. *Hum Brain Mapp*. 2014;35(2):593–615.
65. Lauritzen M. Relationship of spikes, synaptic activity, and local changes of cerebral blood flow. *J Cereb Blood Flow Metab*. 2001;21-(12):1367–1383.
66. Allen G, Buxton RB, Wong EC, Courchesne E. Attentional activation of the cerebellum independent of motor involvement. *Science*. 1997;275(5308):1940–1943.
67. Miall RC, Weir DJ, Wolpert DM, Stein JF. Is the cerebellum a Smith predictor? *J Mot Behav*. 1993;25(3):203–216.
68. Zorzi M, Priftis K, Umiltà C. Neglect disrupts the mental number line. *Nature*. 2002;417(6885):138–139.
69. Breska A, Ivry RB. Double dissociation of single-interval and rhythmic temporal prediction in cerebellar degeneration and Parkinson's disease. *Proc Natl Acad Sci*. 2018;115(48):12283–12288.
70. Kansal K, Yang Z, Fishman AM, *et al*. Structural cerebellar correlates of cognitive and motor dysfunctions in cerebellar degeneration. *Brain*. 2017;140(3):707–720.
71. Hoche F, Guell X, Vangel MG, Sherman JC, Schmahmann JD. The cerebellar cognitive affective/Schmahmann syndrome scale. *Brain*. 2018;141(1):248–270.
72. Timmann D, Daum I. Cerebellar contributions to cognitive functions: A progress report after two decades of research. *Cerebellum*. 2007;6(3):159–162.
73. Molinari M, Petrosini L, Misciagna S, Leggio M. Visuospatial abilities in cerebellar disorders. *J Neurol Neurosurg Psychiatry*. 2004; 75(2):235–240.
74. Brissenden JA, Levin EJ, Osher DE, Halko MA, Somers DC. Functional evidence for a cerebellar node of the dorsal attention network. *J Neurosci*. 2016;36(22):6083–6096.
75. Chen SHA EKH, Ho MHR, Desmond JE. A meta-analysis of cerebellar contributions to higher cognition from PET and fMRI studies: A meta-analysis of cerebellar contributions. *Hum Brain Mapp*. 2014;35(2):593–615.
76. Ziemus B, Baumann O, Luerding R, *et al*. Impaired working-memory after cerebellar infarcts paralleled by changes in BOLD signal of a cortico-cerebellar circuit. *Neuropsychologia*. 2007;45(9):2016–2024.
77. Appollonio IM, Grafman J, Schwartz V, Massaquoi S, Hallett M. Memory in patients with cerebellar degeneration. *Neurology*. 1993;43(8):1536–1536.
78. Spencer RMC, Zelaznik HN, Diedrichsen J, Ivry RB. Disrupted timing of discontinuous but not continuous movements by cerebellar lesions. *Science*. 2003;300(5624):1437–1439.
79. Posner MI, Boies SJ. Components of attention. *Psychol Rev*. 1971; 78(5):391–408.
80. Tsal Y. Movement of attention across the visual field. *J Exp Psychol Hum Percept Perform* 1983;9(4):523–530.
81. Roth MJ, Synofzik M, Lindner A. The cerebellum optimizes perceptual predictions about external sensory events. *Curr Biol*. 2013;23(10):930–935.